Backgrounds without Relativistic Invariance and Holography

Alberto Zaffaroni

Università di Milano-Bicocca

June 14, HATCH 2012

Outline

- Motivations
- Part I. Backgrounds with Non-Relativistic Scale Invariance
- Part II. Supersymmetry on Curved Backgrounds
- Conclusions and Open Problems

Outline

- Motivations
- Part I. Backgrounds with Non-Relativistic Scale Invariance
- Part II. Supersymmetry on Curved Backgrounds
- Conclusions and Open Problems

based on
N. Halmagyi, M. Petrini, A. Z. arXiv:1102.5740
M. Petrini, A. Z. arXiv: 1202.5542
C. Klare, A. Tomasiello, A. Z. arXiv:1205.1062
D.Cassani, C. Klare, D. martelli, A. Tomasiello, A. Z. arXiv:12mm.nnnn

Motivations

- Understanding Different Forms of Holography
- Understanding condensed matter applications using non-relativistic scale invariant duals.
- Understanding 4d/3d gauge theory physics on non-trivial backgrounds.

Lifshitz symmetry

A four-dimensional space-time with Lifshitz symmetry of degree z

$$\mathrm{d}s^2 = R^2 \left(r^{2z}\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{r^2} - r^2\mathrm{d}x^2 - r^2\mathrm{d}y^2 \right)$$

$$(t\,, x\,, y\,, r\,) \to (\lambda^z\, t\,, \lambda\, x\,, \lambda\, y\,, \lambda^{-1}\, r\,)$$

- $z = 1$ corresponds to AdS_4
- $z = 2$ case interesting: susy + existence of central charges
- curvature bounded but tidal forces at the horizon for $z \neq 1$

Lifshitz symmetry

Lifshitz backgrounds can be found as vacua of theories with massive vectors with profile for the gauge field $A_t^\Lambda = r^z A^\Lambda$: [kachru,liu,mulligan;taylor]

$$\mathcal{L} = -\frac{1}{2}\mathcal{R} + \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^\Lambda_{\mu\nu} F^{\mu\nu\,\Sigma} + h_{uv}\nabla^\mu q^u \nabla_\mu q^v - V(q)$$

$$\nabla_\mu q^u = \partial_\mu q^u + k^u{}_\Lambda A^\Lambda{}_\mu$$

At a scalar extremum:

$$h_{uv} k^u_\Lambda k^v_\Sigma A^\Sigma = -\frac{z}{R^2}\mathrm{Im} N_{\Lambda\Sigma} A^\Sigma$$

$$h_{uv} k^u{}_\Lambda k^v{}_\Sigma A^\Lambda A^\Sigma = (z-1)$$

$$V = -\frac{z^2+z+4}{2\,R^2}$$

Schrödinger symmetry

The different space-time

$$\mathrm{d}s^2 = R^2(r^{2z}\mathrm{d}x_+^2 - 2r^2\mathrm{d}x_+\mathrm{d}x_- - \frac{\mathrm{d}r^2}{r^2} - r^2\mathrm{d}x^2)$$

is invariant under the scaling symmetry

$$(x_+\,, x_-\,, x\,, r\,) \rightarrow (\lambda^z\, x_+\,, \lambda^{2-z}\, x_-\,, \lambda\, x\,, \lambda^{-1}\, r\,)$$

- for $z = 2$ we can have a bigger algebra (special conformal transformations)
- it is also solution of Einstein equations coupled to massive fields ($A_+^\Lambda = r^z A^\Lambda$)

Solutions with hyperscaling violation

More general backgrounds [charmousis,goutereaux,kim,kiritsis,myers;dong,harrison,kachru,torroba,wang]

$$ds^2 = u^{-2(1-\frac{\theta}{D})}\Big(- u^{-2(z-1)} dt^2 + \sum_{i=1}^{D} (dx^i)^2 + du^2 \Big) , \qquad u = 1/r$$

The metric is conformal to the Lifschitz space-time but transforms as $ds^2 \to \lambda^{2\theta/D} ds^2$ under the rescaling

$$(t\,, x_i\,, u\,) \to (\lambda^z\, t\,, \lambda\, x_i\,, \lambda\, u\,)$$

- only valid in intermediate range of values for u
- gravity solutions with also running scalars

String theory embedding: non-relativistic solutions

Schrödinger vacua are easily found and, after initial confusion, many Lif_4 vacua have been found in M theory and type II,

[Balasubramanian,Narayan; Donos, Gauntlett; Gregory,Parameswaran,Tasinato,Zavala;Cassani,Faedo; Halmagyi,Petrini,A.Z.]

What we require from an embedding?

- stability (ensured for example by susy? very few non supersymmetric AdS vacua are stable)
- information on the dual theory (stable deformations of AdS vacua?)

Embedding in 4d gauged supergravity: an example

We performed a scan of supersymmetric AdS,Lif_4 and Schr_4 vacua in an $\mathcal{N}=2$ gauged supergravity with one vector multiplet and the universal hypermultiplet [halmagyi,petrini,A.Z.]

$$\mathcal{M}_{\rm SK} \times \mathcal{M}_{\rm Q} = \frac{SU(1,1)}{U(1)} \times \frac{SU(2,1)}{SU(2)\times U(1)}$$

which depending on the type of gaugings (compact/non-compact; electric/magnetic), is a truncation of the $\mathcal{N}=8$ supergravity and other consistent truncations of string theory.

gauging	AdS_4	Schr_4	Lif_4
purely electric or magnetic			X
mixed electric and magnetic	$\mathrm{AdS}_4\times \mathrm{S}^7$ + others	X	

susy Lif_4 always have $z=2$.

Supersymmetric Solutions in 10d

Interestingly, many supersymmetric solutions (2 charges) in 10 d have $z = 2$ and are closely related to existing AdS_5 solutions:

- $\mathrm{AdS}_5\times$ SE $\rightarrow \mathrm{Lif}_4 \times U(1)$ fibration over SE

  [Donos, Gauntlett]

- relativistic solutions in 5-dimensions $\rightarrow$ anisotropic 4d vacua

  [Cassani,Faedo; Petrini,A.Z.]

Supersymmetric Solutions in 10d

More precisely: the condition of supersymmetry for a type IIA metric

$$ds_{10}^2 = -e^{2A_1}dt^2 + e^{2A_2}(dx^2 + dy^2) + \frac{1}{q^2}(d\varphi + \mu)^2 + ds_6^2$$

reduce to those for a relativistic type IIB background

$$ds_{10}^2 = e^{2A}(\eta_{\mu\nu}dx^\mu dx^\nu)^2 + ds_6^2\,, \qquad \mu = 0, \cdots, 3$$

and the identification

$$e^{A_1} = \frac{e^{2A}}{q}\,, \qquad e^{A_2} = e^A\,, \qquad e^{\phi_A} = \frac{e^\phi}{q}$$

plus a set of differential constraints for the fluxes and q. [petrini,A.Z]

Supersymmetric Solutions in 10d

From the standard type IIB vacuum of the form $AdS_5 \times Y$

$$ds^2_{10} = r^2(\eta_{\mu\nu}dx^\mu dx^\nu)^2 + \frac{1}{r^2}(dr^2 + r^2 ds^2_Y)$$

we obtain a Lif_4 solution [gauntlett,donos]

$$ds^2_{10} = -\frac{r^4}{q^2}dt^2 + r^2(dx^2+dy^2) + \frac{dr^2}{r^2} + \frac{1}{q^2}(d\varphi+\mu)^2 + ds^2_Y\,, \qquad e^{-2\phi} = q^2$$

$$4q^2 - \Box_Y q^2 = |d\mu|^2 + \text{fluxes}$$

and with $d\mu$ and internal fluxes which are harmonic forms on Y. Explicit solutions for $Y = T^{1,1}$ (q constant) and some $Y^{p,q}$.

Supersymmetric Solutions in 10d

From the standard type IIB vacuum of the form $AdS_5 \times Y$

$$ds^2_{10} = r^2(\eta_{\mu\nu}dx^\mu dx^\nu)^2 + \frac{1}{r^2}(dr^2 + r^2 ds^2_Y)$$

we also obtain a solution with hyperscaling violation [narayan;dey,roy]

$$ds^2_{10} = -r^6 dt^2 + r^2(dx^2 + dy^2) + r^2 d\varphi^2 + \frac{dr^2}{r^2} + ds^2_Y\,, \qquad e^\phi = r$$

Solution also for $Y = S^5$.

CFTs on Curved spaces

More general vacua of a bulk effective action

$$\mathcal{L} = -\frac{1}{2}\mathcal{R} + \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} F^{\mu\nu\,\Sigma} + \ldots$$

with a metric

$$ds^2_{d+1} = \frac{dr^2}{r^2} + (r^2 ds^2_{M_d} + O(r)) \qquad A^{\Lambda} = A^{\Lambda}_{M_d} + O(1/r)$$

and a gauge fields profile, correspond to CFTs on a d-manifold M_d and a non trivial background field for a set of global currents

$$L_{CFT} + J^{\mu}_{\Lambda} A^{\Lambda}_{\mu}$$

Instead on focusing on scale invariance we can require that some supersymmetry is preserved

Why should we be interested in susy in curved space?

We recently learned more on how to put supersymmetric theories on curved manifolds and how to compute exact quantities. Particularly important objects are:

the partition function of Euclidean supersymmetric theories on spheres.

For example, the partition function of a 3d $\mathcal{N} = 2$ theory on a three sphere:

- it explained the weird scaling $\sim N^{3/2}$ predicted by AdS/CFT.
- it is supposed to decrease along a RG flow (role of a c function in 3D)
- it is maximized at the exact R-symmetry (3d analogous of a-maximization)

CFTs on Curved spaces

Requiring that some supersymmetry is preserved: [klare,tomasiello,A.Z.]

$$\left(\nabla^A_M + \frac{1}{2}\gamma_M + \frac{i}{2}\not{F}\gamma_M\right)\epsilon = 0 \qquad \nabla^A_M \equiv \nabla_M - iA_M$$

Near the boundary:

$$\epsilon = r^{\frac{1}{2}}\epsilon_+ + r^{-\frac{1}{2}}\epsilon_-$$

$$\nabla^A_a \epsilon_+ + \gamma_a \epsilon_- = 0 \qquad \Longrightarrow \qquad \nabla^A_a \epsilon_+ = \frac{1}{d}\gamma_a \not{\nabla}^A \epsilon_+$$

Existence of a Conformal Killing Spinor.

Supersymmetry on Curved spaces

We can understand this by coupling the CFT to background fields of conformal supergravity g_{mn}, ψ_m and A_m:

$$-\frac{1}{2}g_{mn}T^{mn} + A_m J^m + \bar{\psi}_m \mathcal{J}^m$$

In order to preserve some supersymmetry, the gravitino variation must vanish.

$$\delta\psi_m = (\nabla_m - iA_m)\,\epsilon_+ + \gamma_m\epsilon_- = 0$$

$\epsilon_\pm$ parameters for the supersymmetries and the superconformal transformations.

Supersymmetry on Curved spaces

More generally, we may ask when we can put a generic supersymmetric theory on a curved background: coupled it the Poincaré supergravity and set the gravitino variation to zero. [festuccia,seiberg]

Defining $D^A\epsilon_+ \equiv 2iv\epsilon_+$ we can map a CKS to

$$\nabla_a^A\epsilon_+ + \gamma_a\epsilon_- = 0 \qquad \Longrightarrow \qquad \nabla_m\epsilon_+ = -i\left(\frac{1}{2}v^n\gamma_{nm} + (v-a)_m\right)\epsilon_+$$

Condition for coupling a supersymmetric theory to new minimal supergravity $(g_{\mu\nu}, a_\mu, v_\mu)$. Same as the condition for existence of a CKS if $a \equiv A + \frac{3}{2}v$.

When we can have susy in curved space?

Focusing on one preserving one superchange in four dimensions, the conditions for the existence of a CKS are:

- Euclidean: M_4 should be (locally) a complex manifold

  In euclidean we can allow the background fields to be complex (and we must on spheres, for example)

  [klare,tomasiello,A.Z.;dumitruescu,festuccia,seiberg]

- Lorentzian: M_4 should have a null Conformal Killing Vector

$$\nabla_\mu z_\nu + \nabla_\nu z_\mu = \lambda\, g_{\mu\nu} \qquad z_\mu z^\mu = 0$$

  [klare,cassani,martelli,tomasiello,A.Z.]

The background field A_μ is determined in terms of the geometry of the manifold.

String theory embedding: theories on curved backgrounds

Examples can be obtained by solving the supersymmetric conditions

$$\left(\nabla^A_M + \frac{1}{2}\gamma_M + \frac{i}{2}\not{F}\gamma_M\right)\epsilon = 0$$

for a gauged supergravity. Here the field theory interpretation is clear.

- CFTs on spheres: standard AdS_d
- CFTs on 3d squashed spheres: evaluating $\mathcal{N}^{3/2}$ free energies [Martelli,Passias,Sparks]
- Some Lorentzian examples [Gantlett-Gutowski]

In Lorentzian case every supersymmetric 4d metric is the boundary of a 5d gauged sugra vacuum (regular?) [klare,cassani,martelli,tomasiello,A.Z.]

Conclusions

- Many new backgrounds for holography
- Many new phases in field theory, formal and applied
- Many unsolved questions: regularity of the backgrounds, identification of the dual field theories...