Baryonic Popcorn: Holography and Nuclear Matter

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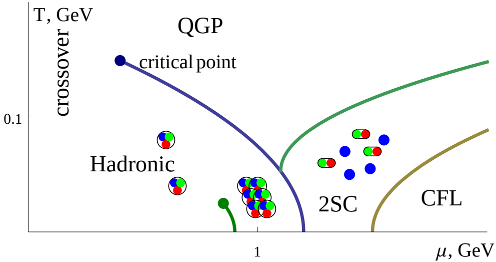
Phase Space of QCD-like Theories

Phase space of QCD



QCD is well understood in two limits: large T ($\mu \to 0$) and large μ ($T \to 0$)

- QGP and CFL phases

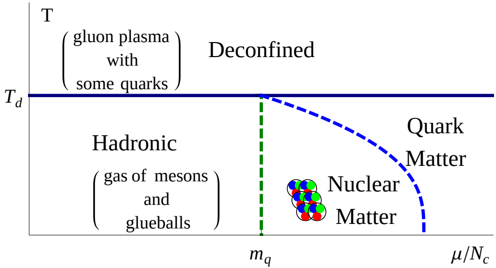
Central part of the diagram is even more interesting

- Nuclear matter (eos, E_b, solid phase), plethora of SC states, χSB ...

Phase Space of QCD-like Theories

Large N_c [McLerran,Pisarski'07]

- Mesons, glueballs are free, mass $\sim \Lambda_{\text{QCD}}$
- Baryons heavy $m_{\text{B}} \sim N_c \Lambda_{\text{QCD}}$ strongly interacting



- T_d = const
- Chiral symmetry restoration may occur separately from deconfining phase transition
- Quarkyonic matter

Skyrmion Matter

Skyrme model [Skyrme'61]

Skyrme proposed to consider baryons as classical solutions of the chiral Lagrangian (low-energy theory of pions)

$$L = \frac{f_\pi^2}{16}\,\mathrm{Tr}\,\partial_\mu U \partial_\mu U^{-1} + \frac{1}{32e^2}\mathrm{Tr}\,\left[\partial_\mu U U^{-1}, \partial_\nu U U^{-1}\right]^2$$

U - $SU(2)$ valued pion field, $U = \sigma(x) + i\pi(x)\cdot\tau$,
f_π - pion decay constant,
e - stabilization term parameter (Skyrme parameter)

Skyrmions are classified by the winding number

$$B = \frac{\epsilon_{ijk}}{24\pi^2}\int_c \mathrm{d}^3x\,\mathrm{Tr}\,\left(\partial_i U U^{-1} \partial_j U U^{-1} \partial_k U U^{-1}\right)$$

Topological lower bound for skyrmion energy: $E_0 = 3\pi^2 f_\pi / e\,|B|$. In the Skyrme model it can never be attained

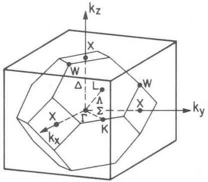
Skyrmion Matter

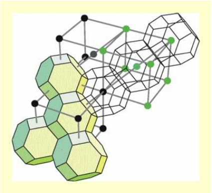
Crystals of skyrmions

Lattices of skyrmions model crystalline phase of large N_c nuclear matter

[Klebanov'85]

- consider single skyrmion solution in a cube. Minimize energy
- phase transitions related to extended symmetries (skyrmions $\rightarrow$ 1/2-skyrmions)





sc $\rightarrow$ 1/2bcc

[Goldhaber,Manton'87]

fcc $\rightarrow$ 1/2sc

[Kugler,Shtrikman'88]

Restoration of chiral symmetry in the half-skyrmion phase

$$\langle \sigma \rangle = 0, \qquad \langle \pi \rangle = 0, \qquad \Longleftarrow \chi\text{SR on average}$$

Skyrmion Matter

Atiyah-Manton ansatz [Atyah,Manton'89]

An approximation to $B = n$ skyrmion can be obtained through

$$U(x_1, x_2, x_3) = \mathcal{P} \exp \left[\int A_4(r, x_4)\, \mathrm{d}x_4 \right]$$

where A_4 is a component of an n-instanton solution in 4d. It does not solve eom, but serves as a good approximation to skyrmion solutions.

Example: the 1-instanton solution is mapped onto the hedgehog

$$U = \mathrm{e}^{iF(r)\, n\cdot\tau}, \qquad F = \pi \left(1 - 1/\sqrt{1 + \frac{\lambda^2}{r^2}} \right)$$

which gives the minimum energy $E = 1.24 E_0$ for $\lambda = 2.11$ (vs 1.23 E_0!)

- Why does this approximation work so well? [Sutcliffe'10]

Sakai-Sugimoto Model

Sakai-Sugimoto model [Sakai,Sugimoto'04]

Gauge sector: N_c D4-branes in type IIA theory. Compactify extra dimensions on S^4. The resulting geometry is dual to the 5d $SU(N_c)$ SYM

Compactify directions along D4 on $\mathcal{M}^{1,3} \times S^1$ with anti-periodic boundary conditions for fermions. This breaks SUSY and introduces a mass scale $M_{KK} \sim \Lambda_{\text{QCD}}$. Pure glue low-energy theory $E \ll M_{KK}$ (Witten's model)

Matter sector: Add N_f probe ($N_f \ll N_c$) D8 and N_f anti-D8 "flavor" branes. D4-D8 strings $\Longrightarrow$ $U(N_f)$ quarks in the low-energy theory $E \ll M_{KK}$

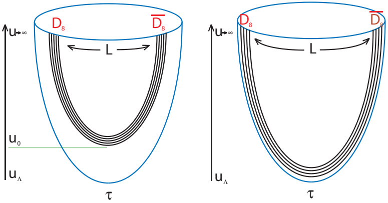
Configuration:

	0	1	2	3	4	5	6	7	8	9
$D4$	○	○	○	○	○					
$D8/\overline{D8}$	○	○	○	○		○	○	○	○	○
	$\mathcal{M}^{1,3}$				S^1	$R+$	S^4			

Sakai-Sugimoto Model

Chiral symmetry breaking [Sakai,Sugimoto'04]

Smooth $S^1 \times R^+$ space must have the shape of a "cigar". Stable embeddings of D8 correspond to U-shaped configurations on the cigar, D8 and $\overline{\text{D8}}$ have to reconnect $\Longrightarrow$ chiral symmetry breaking $U(N_f)_L \times U(N_f)_R \to U(N_f)$



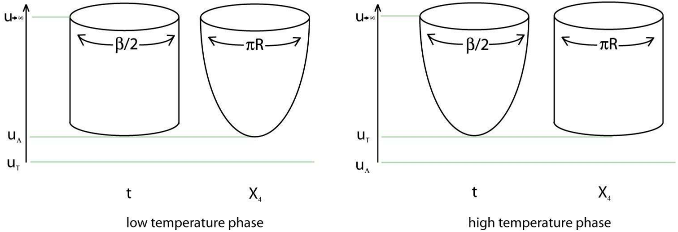
Two dimensionful parameters in the model

- $M_{KK} = 1/R \quad \leftrightarrow \quad U_\Lambda$ – analog of Λ_{QCD}
- $U_0 \quad \leftrightarrow \quad L$ – asymptotic separation of D8-$\overline{\text{D8}}$ – no QCD analog

Sakai-Sugimoto Model

Finite temperature [Aharony,Sonnenschein,Yankielowicz’06]

At finite temperature an extra circle appears in the geometry



Free energy of the left configuration is lower at low temperatures and vice versa. The free energies are equal when $\beta = R$, or $T = M_{KK}$ – deconfinining phase transition (different behavior of Wilson loops)

- $T_d =$ const
- In the high temperature phase there is no need for D8 to reconnect. Restoration of chiral symmetry

Sakai-Sugimoto Model

Baryons [Witten'98]

Baryon vertex (locus for N_c strings) corresponds to a D4 wrapped on S^4. In the Sakai-Sugimoto model such D4 will "dissolve" in D8. Dissolved D4 brane within D8 is an $SU(N_f)$ instanton of the effective theory on D8

$$U(N_f) \text{ gauge fields:} \quad \mathcal{A}_M = A_M + \frac{1}{2}\widehat{A}_M$$

Effective action ($N_f = 2$)

$$S = \int \mathrm{d}^5 x \, \frac{1}{2g_{\mathrm{YM}}^2(x_5)} \,\mathrm{Tr}\, F_{MN}^2 + e \int \widehat{A} \wedge \mathrm{Tr}\, F \wedge F + S[\widehat{A}]$$

Connection to Skyrme model

$$U = P \exp \int A_5 \, \mathrm{d}x_5 \quad \Longrightarrow \quad \begin{array}{c}\text{Chiral}\\ \text{Lagrangian}\end{array} + \begin{array}{c}\text{Skyrme}\\ \text{term}\end{array} + \begin{array}{c}\text{Vector}\\ \text{mesons}\end{array}$$

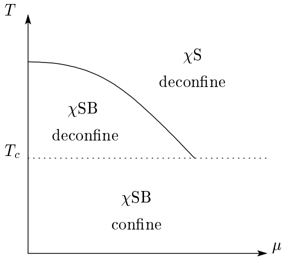
Finite baryon density [Kim,Sin,Zahed'06]

$$x_5 \to \infty: \quad \widehat{A}_0 = \mu + \frac{\rho_{\mathrm{B}}}{x_5} + \ldots \quad \text{Baryon density couples to } \widehat{A}_0$$

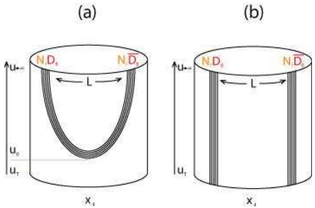
Phase diagram

Restoration of chiral symmetry

In the high temperature phase D8-branes can end on the horizon – restored chiral symmetry. If L is sufficiently small then $T_{\chi\text{SB}} > T_d$



[Aharony,Sonnenschein,Yankielowicz'06]



⟵ Chiral phase transition for $\mu \neq 0$

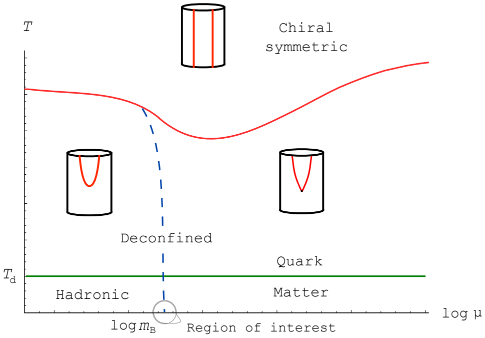
[Horigome,Tanii'06]

Phase diagram

Phase diagram of the Sakai-Sugimoto model [Bergman,Lifschytz,Lippert'07]

Solving instanton equations in curved space is challenging. For large densities one can work in the uniform density approximation (quark liquid)

- $T_d = \text{const}$
- $T_{\chi\text{SB}} \neq T_d$
- 2nd order PT to Quark Matter



- What happens at small densities? How do we describe crystals?

Phase diagram

Nuclear vs Quark Matter

In the large N_c and large $\lambda = g^2_{\rm YM} N_c$ limits the energies scale as

$$m_{\rm B} \sim \lambda N_c, \qquad {\rm K} \sim \frac{1}{m_{\rm B}}, \qquad \Pi \sim N_c$$

- ${\rm K} \ll \Pi$ – at finite density holographic baryons are crystals
- $m_B \gg \Pi$ – baryon interaction is $O(1/\lambda)$ correction to the rest energy.

Size of the baryon $a \sim 1/M_{KK}\sqrt{\lambda}$

- For $\Delta\mu \sim \mu_c$ density is such that the baryons overlap. Overlapping baryons loose their identity: quarks do not know which baryon they belong to

Nuclear matter is confined to a narrow window $\Delta\mu \ll \mu_c$. For larger μ there is a kind of a quark liquid.

Holographic Nuclear Matter

Small instantons

Expand the YM gauge coupling in powers of M_{KK}. For small instantons we can ignore curvature

$$\int \mathrm{d}^4x \, \frac{1}{2g_{\mathrm{YM}}^2} \mathrm{Tr}\, F_{MN}^2 \to \lambda M N_c \int \mathrm{d}^4x \, \left(1 + M^2 x_5^2 + O(M^4 x_5^4)\right) \mathrm{Tr}\, F_{MN}^2$$

Since $a \sim 1/M\sqrt{\lambda}$ this expansion takes the form of $1/\lambda$ expansion. In the leading approximation baryons are non-interacting BPS instantons.

Coulomb energy

$$\frac{N_c}{6\pi} \int \mathrm{d}^4 x \, \widehat{A}_0 \mathrm{Tr}\, F_{MN} \widetilde{F}^{MN} \sim O(N_c \lambda^0)$$

At $O(\lambda^0)$ order baryon is subject to the gravity force from curvature corrections and to Coulomb repulsion. Competition of these two interactions stabilizes the size of the baryon

Holographic Nuclear Matter

Strategy

- Find a relevant (multi-) instanton solution in flat space (≡BPS)

$$O(\lambda): \qquad E_{\rm BPS} = \lambda M N_c \int {\rm d}^4x \,{\rm Tr}\, F_{MN}^2 = \lambda M N_c \times n$$

- Solve for the abelian field

$$\Box \widehat{A}_0 = {\rm Tr}\, F_{MN} \widetilde{F}^{MN}$$

- Compute $1/\lambda$ corrections to the flat space instanton energy

$$O(1): \qquad \delta E = \lambda M^3 N_c \int {\rm d}^4x \, x_5^2 {\rm Tr}\, F_{MN}^2 + \frac{N_c}{6\pi} \int {\rm d}^4x \, \widehat{A}_0 {\rm Tr}\, F_{MN} \widetilde{F}^{MN}$$

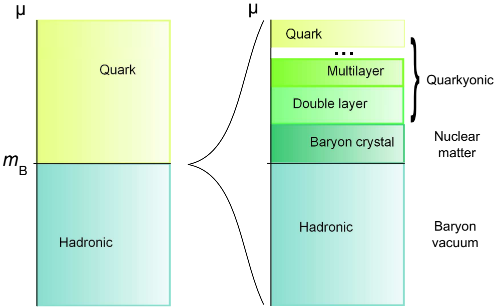
- Minimize the total energy wrt moduli (location, size, orientation)

Holographic Nuclear Matter

Baryonic popcorn [Kaplunovsky,DM,Sonnenschein,'12]

Dynamics of holographic baryons (instantons) is a competition of the gravity force from the curvature of the holographic dimension and Coulomb-like repulsion (at short distances).

- At arbitrary low density instantons must be aligned in 3D
- At large density one expect instantons to pile up in 4D



Connection to skyrmion-half-skyrmion transitions? [Rho,Sin,Zahed'09]

Holographic Nuclear Matter

Point charge model

In the point-charge approximation for baryons and a 1d lattice

$$\mathrm{Tr}\, F_{MN}\widetilde{F}^{MN} \to \sum_n \delta^{(4)}(x_4 - n \cdot d)$$

The energy of such a configuration computed per baryon

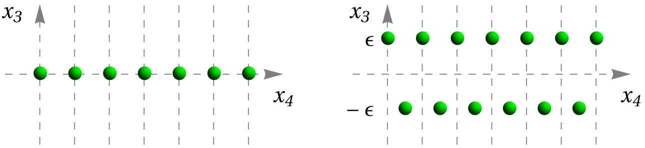
$$E = \lambda M N_c \left(1 + M^2 z^2 + M'^2 (x_1^2 + x_2^2)\right) + \frac{N_c}{4\lambda M} \sum_n \frac{1}{(nd)^2}$$

Curvature pins baryons to $x_1 = x_2 = x_5 = 0$. If we squeeze the chain, at some density Coulomb repulsion will push baryons up in the holographic dimension

Crystals of Holographic Baryons

Zigzag instability [Kaplunovsky,DM,Sonnenschein'12]

The leading instability must correspond to nearest neighbors displacing in opposite directions transverse to the chain – zigzag



The energy of the zigzag with amplitude ϵ per baryon

$$E = E_0 + N_c \lambda M^3 \epsilon^2 + \frac{N_c}{\lambda M}\left(-\frac{\pi^4\epsilon^2}{48 d^4} + \frac{\pi^6\epsilon^4}{120 d^6} + O(\epsilon^6)\right)$$

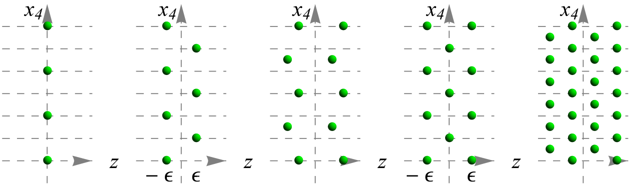
ϵ^2 -term changes sign at $d_c = \pi/(2\cdot 3^{1/4} M\sqrt{\lambda}) \Rightarrow$ second order PT

$$\langle\epsilon\rangle \simeq \pm\frac{\sqrt{5}}{\pi}\sqrt{d_c(d_c-d)}$$

Crystals of Holographic Baryons

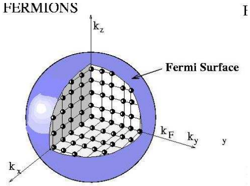
Cascade of transitions [Kaplunovsky,DM,Sonnenschein'12]

When the density is increased the chain becomes thicker and thicker



It is straightforward to generalize this story to the case of 3D lattices

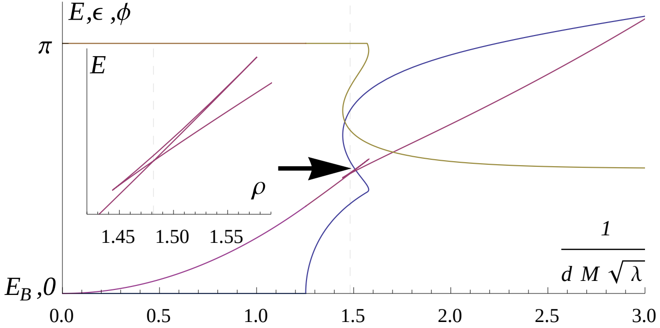
What is the interpretation of the thick chain?
1/2-skyrmion phase? Quarkyonic phase?



Holographic Nuclear Matter

1D instanton chain [Kaplunovsky,DM,Sonnenschein'12]

Finite size instanton configurations require solutions of complicated ADHM equations. (New solutions!) The phase space of the full instanton solutions is even richer.



Conclusions

Holography allows to probe hadron physics in the regimes not accessible from the first principle approach. Various features of the hadron physics are nicely incorporated (χSB, baryons, Skyrme theory,...) The price to pay is the unconventional limit of $N_c \to \infty$ and $\lambda \to \infty$. For finite T and μ_{B}

- holography gives results consistent with general expectations at large N_c;
- provides an interesting geometrical interpretation of χSB. Separates the χSB transition from confinement;
- provides tools to analyze nuclear matter phase

In the nuclear matter phase

- piling up of lattices in 4D
- rich phase space of lattice configurations

Conclusions

Future directions

- Interpretation of the 4D nature of the holographic baryon lattices. Quarkyonic phase?
- Analysis of 3D instanton lattices. New instanton solutions
- Correspondence between the phase space of holographic baryon and Skyrme lattices. Skyrmion-half-skyrmion phase transitions, chiral symmetry

Hadronic physics: a challenge to holography

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