Baryon as a domain wall in holographic QCD

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Baryon

Baryon is a colorless object in quantum chromodynamics, carrying unit baryon charge:

$$\langle B|J_0|B\rangle = 1$$

where J_μ is a current of vector $U(1)$ subgroup in chiral $U(N_f)_L \times U(N_f)_R$.

Baryons have a mass of the QCD scale Λ_{QCD} and, apart of baryonic, carry the axial charge

$$g_A \simeq 1.2$$

Skyrmion

First attempt of describing a baryon as a topological object was performed by Skyrme (*Proc.Roy.Soc.Lond. A260 (1961), 127*)

In a chiral perturbation theory, being the nonlinear sigma model of $U = \exp(\pi^a t^a)$, the baryon was shown to be a skyrmion, the classical nonlinear solution

$$U_0(\mathbf{x}) = \exp\left(f(r) \frac{x^a t^a}{r} \right)$$

with nonzero winding number.

$$B = \int \frac{1}{24\pi} \epsilon_{ijk} \langle (U\partial_i U^\dagger)(U\partial_j U^\dagger)(U\partial_k U^\dagger) \rangle$$

5D domain wall

The connection of skyrmion with instanton was figured out by Atiah and Manton (*Phys.Lett. B222 (1989) 438-442*)

$$U(\mathbf{x}) = \mathcal{P} \exp \int dx^4 \; A_4(\mathbf{x}, x_4)$$

and the uplift to 5 dimensions was performed by Tong et al. (*Phys.Rev.Lett. 95 (2005) 252003*).

The skyrmion was shown to be a holonomy of instanton, taking refuge inside the domain wall from the broken gauge symmetry of the bulk

Holographic Baryons

In the holographic models baryons also appeared to be related to instantons. Considering the Chern-Simons term in 5D action Sakai et al. (*Prog.Theor.Phys. 117 (2007) 1157*) figured out, that it sources the baryon $U(1)$ current $\hat{A}_0$

$$S_{CS} = \int d^4xdz \, \frac{N_c}{24\pi^2} \, \epsilon_{MNPQ} \, \hat{A}_0 tr\left(F_{MN}F_{PQ}\right) + \dots$$

And the baryon charge coincides with the topological charge of the field configuration in 4D spacelike slice of the bulk space.

Holographic Baryons

Holographic baryons were extensively studied and they appeared to have some problems:

- The holographic instantons tend to zero size, becoming singular.
- Due to the curvature of space they tend to fall on the IR boundary of the model.
- The axial charge of the baryon is hard to define.

Dyonic instanton

Meanwhile an interesting topological solution in 5D was found by Lambert and Tong (*Phys.Lett. B462 (1999) 89-94*), the dyonic instanton:

$$A_\mu = \frac{2}{g}\frac{\rho^2}{x^2(x^2+\rho^2)}\eta^a_{\mu\nu}x_\nu\frac{\sigma^a}{2}, \qquad \phi = v\frac{x^2}{x^2+\rho^2}\frac{\sigma^3}{2}$$

Thanks to the scalar field, which brakes gauge symmetry by its vacuum expectation value, it possess a nonabelian electric charge, which stabilizes the radius of the 4D instanton.

$$\rho^2 = \frac{1}{4\pi^2}\frac{Q}{v}$$

Holographic model

In AdS/QCD the action contains two $SU(2)$ gauge fields, related to left and right currents

$$S = \int d^3x \; dt \; dz \left\{ \frac{1}{z} \left(-\frac{1}{4g_5^2} \right) (F_L^2 + F_R^2) \right\}$$

The baryon in this setup was studied by Pomarol and Wulzer (*Nucl.Phys. B809 (2009) 347*) and it was shown, that the baryon number here equals

$$B = \frac{1}{32\pi^2} \int d^3x \int_{\epsilon}^{z_m} dz \; \left\langle F_L \tilde{F}_L - F_R \tilde{F}_R \right\rangle$$

Holographic model

Including the bifundamental scalar field X, dual to the scalar quark current $\langle \bar{q} q \rangle$ one gets

$$S = \int d^3x \; dt \; dz \left\{ \frac{1}{z} \left(-\frac{1}{4g_5^2} \right) (F_L^2 + F_R^2) + \Lambda^2 \left[\frac{1}{z^3} (DX)^2 + \frac{3}{z^5} \, |X|^2 \right] \right\}$$

$$D_\mu X = \partial_\mu X - i A_L X + i X A_R$$

One immediately recognizes the scalar with VEV, needed for the dyonic instanton solution

$$X_0 \sim \sigma z^3$$

Cylindrical ansatz

To study the solution with scalar field we adopt the "cylindrical ansatz" for gauge fields

$$A_j^a = \frac{1+\varphi_2}{r}\epsilon_{jak}\frac{x_k}{r} + \frac{\varphi_1}{r}\left(\delta_{ja} - \frac{x_j x_a}{r^2}\right) + A_r \frac{x_j x_a}{r^2}$$
$$A_z^a = A_z \frac{x_a}{r}$$

and for scalar field

$$X = \chi_1 \frac{\mathbf{1}}{2} + i\chi_2 \frac{\tau^a x^a}{r}$$

Cylindrical ansatz

Moreover, the P-parity conditions

$$A_i^L(x,z) = -A_i^R(-x,z), \qquad L_z(x,z) = R_z(-x,z)$$

force additional constraints on the new fields

$$\varphi_1^L = -\varphi_1^R, \qquad \varphi_2^L = \varphi_2^R, \qquad A_r^L = -A_r^R, \qquad A_z^L = -A_z^R,$$

2D action

The action for complex fields

$$\boldsymbol{\varphi} = \varphi_1 + i\varphi_2 \equiv \varphi \; e^{i\alpha},$$
$$\boldsymbol{\chi} = \chi_0 + i\chi_1 \equiv \chi \; e^{i\beta},$$

looks like

$$S = \int dt\; 4\pi \int drdz \Bigg\{ -\frac{1}{2g_5^2}\Big[\frac{2}{z}|D_a\boldsymbol{\varphi}|^2 + \frac{1}{2}(F_{ab})^2 + \frac{1}{r^4}(1-|\boldsymbol{\varphi}|^2)^2\Big] -\Lambda^2\Big[\frac{r^2}{2z^3}|D_a\boldsymbol{\chi}|^2 + \frac{1}{z^3}\chi^2\varphi^2\;\cos(\alpha-\beta)^2\Big] +\Lambda^2\;\frac{3r^2}{2z^5}|\boldsymbol{\chi}|^2\Bigg\},$$

and has two interesting potential terms

Topological charge

The first potential

$$\frac{1}{r^4}(1-|\varphi|^2)^2$$

defines the vacua

$$\varphi\Big|_{vac} = e^{i\alpha}.$$

The solution, interpolating between vacua with different α has a topological charge

$$B = \frac{1}{2\pi}\int dr \int_{\epsilon}^{z_m} dz\ \epsilon_{ab}\Big(\partial_a(-i\varphi D_b\overline{\varphi} - h.c.) + F_{ab}\Big),$$

where $\partial_a(-i\varphi D_b\overline{\varphi} - h.c.) = 2\partial_a(\varphi^2\partial_b\alpha)$

Second charge

The second potential

$$\frac{1}{z^3}\chi^2\varphi^2 \ \cos(\alpha-\beta)^2$$

defines a discrete set of vacua

$$\gamma = \alpha - \beta - \frac{Pi}{2} = \pi n, \quad n \in \mathbb{Z}.$$

The solution, interpolating between vacua with different γ should have second topological charge.

Axial current

We can guess what is the nature of this charge by looking at the phase β.
The X field is a source to the axial current, namely

$$J_\mu^A \sim i(\partial_\mu X X^\dagger - X \partial_\mu X^\dagger).$$

In the 2D fields notation, this expression looks as

$$J_r^A \sim 2\chi^2 \, \partial_r \beta(r, z).$$

Consequently the solution with changing β couples to the axial current, thus it has an axial charge $g_A \neq 0$.

Possibility of dyonic solution

In what fallows, we study the boundary values of the 2D fields, which respect two requirements:

- The action of the solution under consideration is finite
- The solutions for the fields on the boundaries are not singular

Boundary values: $r = 0$

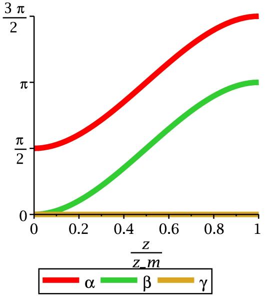
At $r = 0$ $\frac{1}{r^4}(1 - |\varphi|^2)^2$ diverges, consequently we impose

$$\varphi(r, z)\Big|_{r=0} = 1.$$

Moreover, singular equations of motion require

$$\gamma(0, z) = 0$$

But α and β are unconstrained



Boundary values: $z = 0$

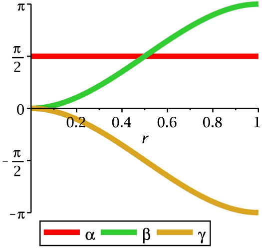
At $z = 0$ the metric diverges, so for the finiteness we need

$$\varphi(r,z)\Big|_{z=0} = 1, \qquad \chi(r,z)\Big|_{z=0} \sim \sigma z^3, \qquad A_r(r,z)\Big|_{z=0} \sim z^2$$

And for regularity of the solution

$$\partial_r \alpha(r,\epsilon) = -A_r(r,\epsilon) \sim \epsilon^2$$

β and γ – unconstrained

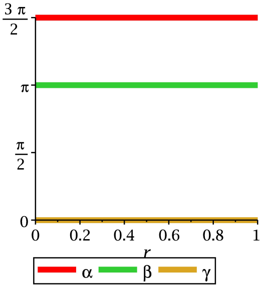


Boundary values: $z = z_m$

In the hard-wall model one usually requires vanishing of the action on the IR-wall.

$$\varphi(r,z)\Big|_{z=z_m} = 1, \quad \chi(r,z)\Big|_{z=z_m} = const, \quad A_r(r,z)\Big|_{z=z_m} = const$$

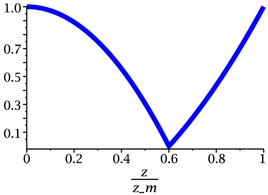
$$\partial_r\alpha(r,z_m) = \partial_r\beta(r,z_m) = 0$$
$$\partial_r\gamma(r,z_m) = 0$$



Boundary values: $r = \infty$

The $r = \infty$ is the most interesting, because though $\frac{1}{z^3}\chi^2\varphi^2 \cos(\alpha-\beta)^2$ is singular, the potential $\frac{1}{r^4}(1-|\varphi|^2)^2$ is irrelevant. So we can find nontrivial boundary value for φ.

$$\varphi(r,z)\Big|_{r\to\infty} = \theta(z_0 - z)\left(1 - \frac{z^2}{z_0^2}\right) + \theta(z - z_0)\left(\frac{z^2 - z_0^2}{1 - z_0^2}\right)$$



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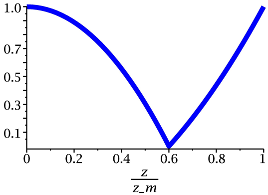
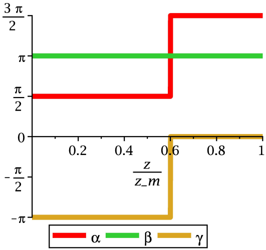


This solution allows γ to jump at certain point z_0, where ϕ vanishes.

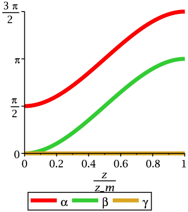
$$\gamma(r,z)\Big|_{r\to\infty} = -\pi + \theta(z - z_0)\,\pi,$$
$$\beta(r,z)\Big|_{r\to\infty} = const$$



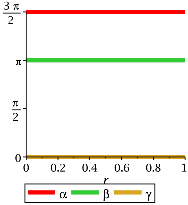
Overview of the solution

At the end of the day we got the nontrivial allowed boundary values for the solution, carrying two topological charges.

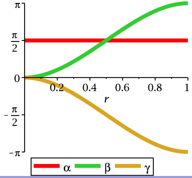
$r = 0$



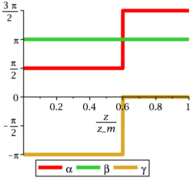
$z = z_m$



$z = 0$



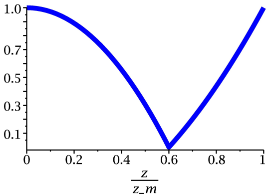
$r \to \infty$



Overview of the solution

Because of the form of the boundary value of ϕ, the position of the solution can not fall on the IR wall

$$z_0 \neq z_m$$

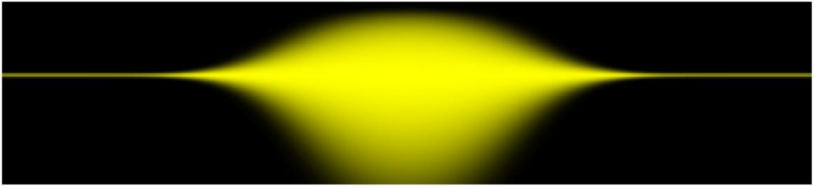


Moreover, one can argue, that the position of the solution z_0 is governed by the value of the vacuum expectation value of the scalar field σ dual to the chiral condensate, which is the only tunable parameter of the model. That leads to the holographic proof to the Ioffe relation:

$$M_B \sim \langle \bar{q} q \rangle^{-1/3}$$

Overview of the solution

The discussed solution has a form of domain wall, whose thickness, while being zero at radial infinity, rises closer to the core, covering the whole distance between holographic boundaries in the center of instanton.



Artist view: the energy density of holographic dyonic instanton

Conclusion

In this work we show a possibility to construct the "dyonic instanton" solution in AdS/QCD. The solution

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- Has a mass related to the chiral condensate

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In this work we show a possibility to construct the "dyonic instanton" solution in AdS/QCD. The solution

- Has nonzero baryon charge
- Has nonzero axial charge
- Can not fall on the IR boundary, thus it is can not be singular by construction
- Has a mass related to the chiral condensate
- Has a peculiar form of flying saucer

Thank you for your attention!