# The tensor responce on the external magnetic field in the holographic QCD

P. Kopnin, A. Krikun, A. Vainshtein and A.G.
1201.2039

INR, June 11

# Plan of the talk

1. Introduction.Magnetic susceptiibility of the condensate. Son-Yamamoto telation.

2. SY relation in the ChPT theory. Attempt of derivation.

3. Holographic scalar-tensor mixing in the magnetic field via anomaly.

4. Weak and strong magnetic field and susceptibility

5. Conclusion

The magnetic ssceptibility measures the induced tensor current in the chirally broken QCD vacuum in the external magnetic field (Ioffe-Smilga 83)

Exact value

In pion dominance model

Vainshtein 04

$$\chi_v = -\frac{N_c}{4\pi^2 f_\pi^2}$$

It is anomalous object amounted from Chern-Simons term Krikun-A.G. 09

$$\langle \bar{q}\sigma_{\mu\nu} q\rangle_F = \chi \langle \bar{q}q\rangle F_{\nu\mu}$$

# Son-Yamamoto relation 10

$$w_T(Q^2) = \frac{N_C}{Q^2} - \frac{N_C}{f_\pi^2}\left[\Pi_A(Q^2) - \Pi_V(Q^2)\right]$$

With the definitions

$$\langle V_\mu A_\nu \rangle_{\tilde{F}} = \frac{1}{4\pi^2}\left[w_T(q^2)(-q^2\tilde{F}_{\mu\nu} + q_\nu q^\sigma \tilde{F}_{\mu\sigma} - q_\mu q^\sigma \tilde{F}_{\nu\sigma}) + w_L(q^2)\, q_\nu q^\sigma \tilde{F}_{\mu\sigma}\right],$$

$$\frac{1}{2}\operatorname{Tr}(QVA)\,\langle V_\mu A_\nu \rangle_F \equiv \int d^4x\, e^{iqx} \langle T\{V_\mu(x) A_\nu(0)\}\rangle_F\,,$$

$$\frac{1}{2}\operatorname{Tr}(VV)\,\Pi_V(Q^2)(q_\mu q_\nu - g_{\mu\nu}q^2) \equiv \int d^4x\, e^{iqx} \langle T\{V_\mu(x) V_\nu(0)\}\rangle_0\,,$$

$$\frac{1}{2}\operatorname{Tr}(AA)\,\Pi_A(Q^2)(q_\mu q_\nu - g_{\mu\nu}q^2) \equiv \int d^4x\, e^{iqx} \langle T\{A_\mu(x) A_\nu(0)\}\rangle_0\,,$$

At the large virtualiity it yields exactly the Vainshtein relation for susceptibility of the condensate

What could be said about the SY relation at small virtualities?

Proper approach -ChPT

Matching

1. Costant  terms (independent on momenta)

2. Chiral infrared logs

## Constant terms matching condition (de Rafael et al 10)

$$L_{10} = -4\pi^2 C_{22}$$

## Matching of chiral logs

$$\Pi_V^{\text{chir}}(Q^2 \to 0) = c \log Q^2 \,, \qquad c = -\frac{1}{48\pi^2} \,.$$

$$S^{3\pi}_{WZW} = -\frac{N_c}{24\pi^2} \int \text{Tr}\, A (dU^{-1}U)^3 \to -\frac{iN_c}{24\pi^2 f_\pi^3} \int d^4x \, \text{Tr}\, \tilde{F}^{\gamma\delta} \pi \, \partial_\gamma \pi \, \partial_\delta \pi \,.$$

$$w_T^{\text{chir}} = c_1 \log Q^2 \,, \qquad c_1 = \frac{N_c}{f_\pi^2} c \,,$$

OK!

Let us introduce the tensor source in the ChPT

$$\delta L_1 = B_{\mu\nu}\,\bar{q}\,B\,\sigma^{\mu\nu}\,q \equiv i\,\tilde{B}_{\mu\nu}\,\bar{q}\,B\sigma^{\mu\nu}\,\gamma_5 q$$

Upon the integration over the quark loops the mixed anomalous term emerges

$$\delta L_{WZW} = -\frac{1}{2}\,\chi\,\langle\bar{q}q\rangle\,B_{\mu\nu}\,F^{\mu\nu}\,\mathrm{Tr}\,(U+U^{\dagger})BQ\,,$$

The susceptibility — coefficient in front of the mixed anomaly

# Some interesting effects of the mixed anomaly

$$\langle J_\nu \rangle_B = \frac{\delta S_{WZW}}{\delta A_\nu}$$

# EM current in the chirally broken vacuum in the external field

$$\langle J_\nu \rangle_B = \frac{1}{2} \chi \langle \bar{q}q \rangle \partial^\mu \left[ B_{\mu\nu} \mathrm{Tr}\,(U + U^\dagger) BQ \right].$$

# Anomalous tensor-2 pions vertex

$$\langle 0 | \bar{q} \sigma_{\mu\nu} q | \pi^0 \pi^0 \rangle = \frac{1}{3 f_\pi^2} \chi \langle \bar{q}q \rangle F_{\mu\nu} \; .$$

No field theory derivation of SY relation.How to get it?

The relation between two-point and three point correlators of currents. In the bulk relation between different derivatives of the action with respect to «coordinates>.

Hamiltonian dynamics in the radial AdS time! It was identified as the RG flows (Gerasimov-unpublished,Verlinde-Verlinde-de Boer 98-99)

In the Hamiltonian dynamics there are two independent relations

1 Gauss law constraint
2. Hamilton-Jacobi equation

Modification of the canonical momentum due to the CS term

$$\Pi_L = E_L + A_L F_L\,, \qquad \Pi_R = E_R - A_L F_L\,,$$

$$J_\mu = \frac{\delta S}{\delta A} \qquad \Pi = \frac{\delta S}{\delta x}\,,$$

Gauss law in D=5 coincides with the anomaly equation in D=4!

RG Hamiltonian $\left(\frac{\delta S}{\delta A} - AF\right)^2 + F_{ij}^2$

The HJ equation H=E due to the modification of canonical momentum mixes the terms with different nmber of derivatives. Upon the additional derivatives very close to SY(not the same)

# Consider the effect of mixed anomalous term in holography

$$ds^2 = \frac{\ell^2}{z^2}\left(-dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right), \qquad 0 \le z \le z_m,$$

$$\bar{q}_{R\bar{f}}\, q_L^f \leftrightarrow X_{\bar{f}}^f, \qquad \bar{q}_{Rg}\gamma_\mu q_R^{\bar{f}} \leftrightarrow R_{\mu\, g}^{\bar{f}},$$

$$\bar{q}_{R\bar{f}}\,\sigma_{\mu\nu} q_L^f \leftrightarrow B_{\mu\nu\,\bar{f}}^f, \qquad \bar{q}_{L\,g}\gamma_\mu q_L^f \leftrightarrow L_{\mu\, g}^f,$$

Both scalar and tensor interact with the gauge field

$$DX = dX - iLX + iXR,\ H = DB = dB - iL \wedge B + iB \wedge R.$$

Tensor field in the bulk (Karch et al 10)

$$S = \int d^5x \sqrt{-g}\, \mathrm{Tr}\left\{ -\frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right) + g_X^2\left(|DX|^2 - m_X^2|X|^2\right) \right.$$
$$\left. - 2g_B\left(\frac{i}{6}\left(B \wedge H^+ - B^+ \wedge H\right) + m_B|B|^2\right) + \frac{\lambda}{2}\left(X^+ F_L B + B F_R X^+ + \mathrm{c.c.}\right)\right\}.$$

The holographic action involves XB mixing in the magnetic field

$$X(z) = \frac{1}{2}\left(mz + \frac{1}{g_X^2}\langle\bar{q}q\rangle z^3\right) \times \mathbf{1}_{N_f \times N_f}\,.$$

The chiral symmetry breaking is encoded in the solution

$$X = \frac{X_+ + iX_-}{2}; \quad B_{MN} = \frac{(B_+ + iB_-)_{MN}}{\sqrt{2}}.$$

It is useful to consider the following decomposition

$$\bar{q}q \leftrightarrow X_+ , \qquad \frac{1}{\sqrt{2}}\bar{q}\sigma_{\mu\nu}q \leftrightarrow B_{+\mu\nu} ,$$

$$i\bar{q}\gamma_5 q \leftrightarrow X_- , \qquad \frac{i}{\sqrt{2}}\bar{q}\gamma_5\sigma_{\mu\nu}q \leftrightarrow B_{-\mu\nu} ,$$

$$\bar{q}\gamma_\mu q \leftrightarrow V_\mu .$$

$$\mathcal{S} = \int d^5x \sqrt{-g}\, \mathrm{Tr} \Big\{ -\frac{1}{4g_5^2} F_V^2 + \frac{g_B}{3} \epsilon^{MNPQR} (B_{-MN} H_{+PQR} - B_{+MN} H_{-PQR})$$
$$+ \sum_{+,-} \Big[ - g_B m_B B_{\pm MN} B^{\pm MN} + \frac{g_X^2}{4} (\partial_M X_\pm \partial^M X_\pm - m_X^2 X_\pm^2) + \frac{\lambda}{2} X_\pm (F_V)_{MN} B_\mp^{MN} \Big] \Big\}.$$

Now the equations of motion read as

$$\pm z \epsilon^{\mu\nu\lambda\rho} H_{\pm z\lambda\rho} + 2 B_\mp^{\mu\nu} = \frac{\lambda}{4g_B} X_\mp F_V^{\mu\nu},$$
$$\pm \frac{z}{3} \epsilon^{\mu\lambda\rho\sigma} H_{\pm\lambda\rho\sigma} + 2 B_\mp^{\mu z} = \frac{\lambda}{4g_B} X_\mp F_V^{\mu z}$$

$$z \partial_z \left( z H_\pm^{z\alpha\beta} \right) + B_\pm^{\alpha\beta} + z^2 \partial_\mu H_\pm^{\mu\alpha\beta} = \frac{\lambda}{8 g_B} \left[ X_\pm F_V^{\alpha\beta} \pm z \partial_z X_\mp \tilde{F}_V^{\alpha\beta} \right],$$
$$z^2 \partial_\lambda H_\pm^{\lambda\mu z} + B_\pm^{\mu z} = \pm \frac{\lambda}{2 g_B} z \partial_\lambda \left( X_\mp \tilde{F}_V^{\mu\lambda} \right),$$
$$\partial_z \left( \frac{1}{z^3} \partial_z X_\pm \right) + \frac{3}{z^5} X_\pm - \frac{1}{z^3} \partial_\mu \partial^\mu X_\pm = -\frac{\lambda}{g_X^2} \frac{1}{z} (F_V)_{\mu\nu} B_\mp^{\mu\nu}.$$

With the following solutions

$$X_+ + iX_- = z^2 f_X(qz) e^{ikx_3 - i\omega t};$$

$$(B_+ + iB_-)_{12} = \frac{g_X}{4\sqrt{g_B}} f_B(qz) e^{ikx_3 - i\omega t},$$

Fixed by boundary conditions

$$f_X(qz) = C_1 \mathcal{J}_1(\sqrt{1+\beta}qz) + C_2 \mathcal{J}_1(\sqrt{1-\beta}qz) + C_3 \mathcal{Y}_1(\sqrt{1+\beta}qz) + C_4 \mathcal{Y}_1(\sqrt{1-\beta}qz);$$

$$f_B(qz) = C_1 \mathcal{J}_1(\sqrt{1+\beta}qz) - C_2 \mathcal{J}_1(\sqrt{1-\beta}qz) + C_3 \mathcal{Y}_1(\sqrt{1+\beta}qz) - C_4 \mathcal{Y}_1(\sqrt{1-\beta}qz),$$

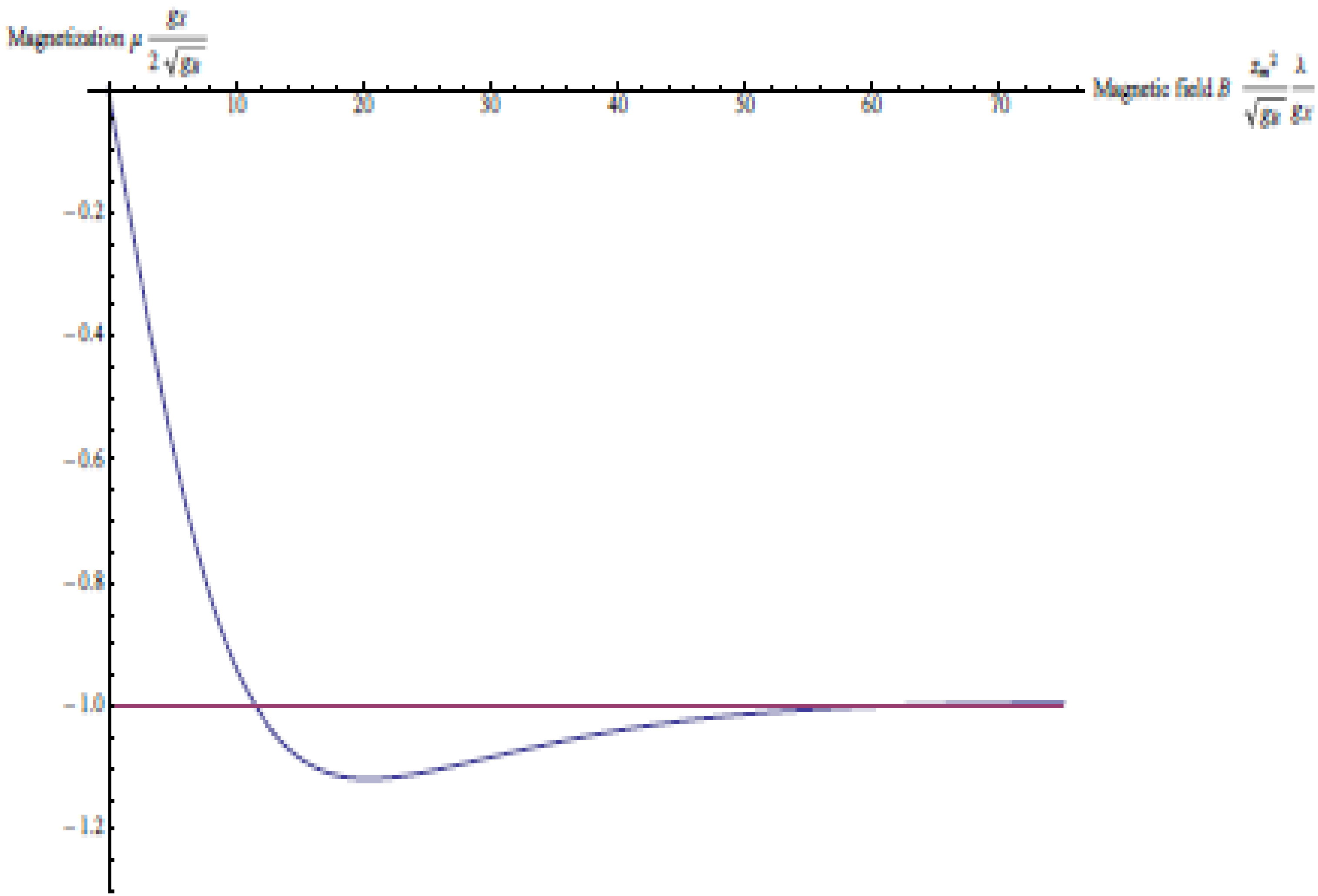
where $\beta = \frac{|\lambda|}{2g_X\sqrt{g_B}} |\mathbf{B}/q^2|$ and $q^2 = \omega^2 - k^2$ :

$$\frac{C_1}{C_2} = \frac{\mathcal{J}_1(\sqrt{1-\beta}\, q z_{\rm m}) + \sqrt{1-\beta}\, q z_{\rm m} \mathcal{J}_1'(\sqrt{1-\beta}\, q z_{\rm m})}{\mathcal{J}_1(\sqrt{1+\beta}\, q z_{\rm m}) + \sqrt{1+\beta}\, q z_{\rm m} \mathcal{J}_1'(\sqrt{1+\beta}\, q z_{\rm m})}.$$

Asymtotics of solution provides the physical quantities

$$\langle \bar{q}\sigma_{12} q \rangle \propto 8 g_B \frac{g_X}{4\sqrt{g_B}} \lim_{z\to 0} \frac{f_B(qz)}{z}; \qquad \langle \bar{q} q \rangle \propto g_X^2 \lim_{z\to 0} \frac{f_X(qz)}{z},$$

$$\mu(\mathrm{B}; q) = \frac{\langle \bar{q}\sigma_{12} q \rangle}{\langle \bar{q} q \rangle} = \frac{2\sqrt{g_B}}{g_X} \lim_{z\to 0} \frac{f_B(qz; \mathrm{B})}{f_X(qz; \mathrm{B})}.$$



Magnetization $\mu \frac{gr}{2\sqrt{gu}}$
Magnetic field $B \frac{z_m^2}{\sqrt{gu}} \frac{\lambda}{gr}$
10
20
30
40
50
60
70
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2

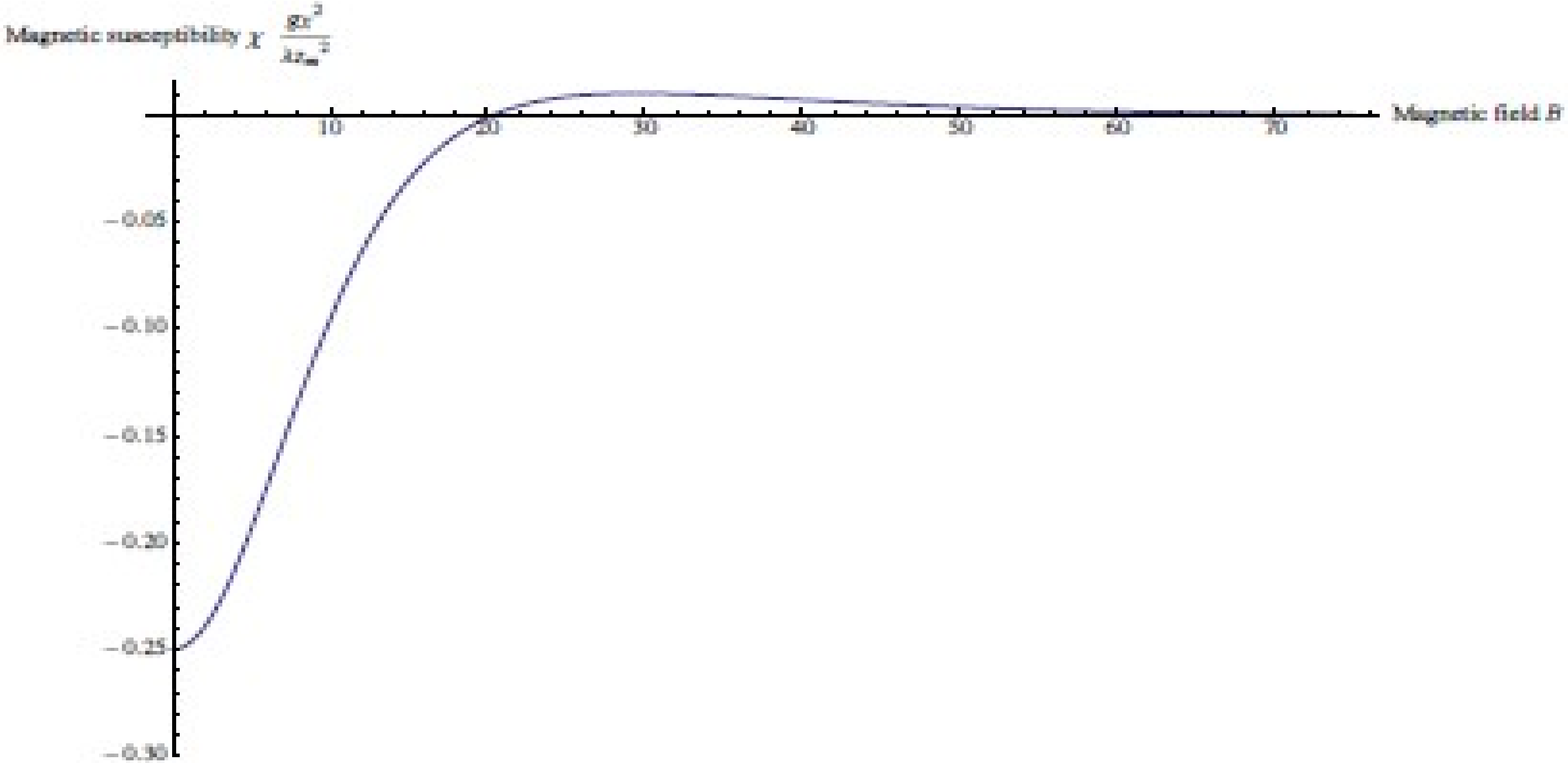



FIG. 2. Magnetic susceptibility of the quark condensate $\chi(\mathbf{B})$

and

$$\chi(\mathbf{B}) = -\frac{|\lambda|}{g_X^2}\frac{z_m^2}{4}\left(1 - \frac{1}{96}\frac{\lambda^2}{g_X^2 g_B}\mathbf{B}^2 z_m^4 + \mathcal{O}\left(\mathbf{B}^4 z_m^8\right)\right), \qquad \mathbf{B} \to 0.$$

$$\mu(\infty) = 1/\sqrt{3}\,, \qquad \chi(0) = -\frac{z_m^2}{72}\,.$$

Theory

$$\chi \sim -3.15 \pm 0.30 \text{ GeV}^{-2},$$

Sum rules

$$\chi \sim -8.9 \text{ GeV}^{-2}.$$

Vainshtein

$$\chi \sim -11.5 \text{ GeV}^{-2}$$

VVA in holography

Different values from different experiments

# Conclusion

1- SY relation is valid in the IR region.

2. SY relation is close to the Hamilton-Jacobi equation in the RG AdS time but needs more analysis. Ward identity for the yet unknown symmetry?

3. Interesting nonperturbative scalar-tensor mixing In the external magnetic field.

4. Tensor field in the bulk in the holographic QCD yields the fixed value of susceptibility(small) and saturation of magnetization at the strong field

5. Stringy degrees of freedom?Stringy current?