Massive fields in AdS and computation of two-point correlation functions via AdS/CFT

R.R. Metsaev

Lebedev Institute

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Plan

1) **Introduction**

2) **Modified (Lorentz) de Donder gauge and computation of two-point functions from AdS**

3) **New (gauge invariant) approach to CFT**

General setup of gravity/gauge theory duality

$$\mathbf{S_{AdS}(\Phi)}$$

$\Phi = \phi$ scalar

ϕ^A vector

ϕ^{AB} tensor

$\phi^{A_1 \dots A_s}$ arbitrary spin

fields in AdS space

AdS_{d+1}

$$ds^2 = \frac{R^2}{z^2}(dx^a dx^a + dz dz)$$

$\mathbf{x^a}$ boundary flat coordinates

$\mathbf{z}$ radial coordinate

$R = 1$

$$\frac{\delta S_{AdS}}{\delta \Phi} = 0$$

$$\Phi(x, z) \sim z^{\Delta} \Phi_{\mathrm{cur}}(x)$$

$$\Phi(x, z) \sim z^{d-\Delta} \Phi_{\mathrm{sh}}(x)$$

$$\Delta = \frac{d}{2} + \sqrt{m^2 + (s + \frac{d-4}{2})^2}$$

Use solution corresponding $\boldsymbol{\Phi}_{\mathrm{sh}}$

$$S_{AdS}(\boldsymbol{\Phi}) \equiv S_{\mathrm{eff}}(\boldsymbol{\Phi}_{\mathrm{sh}})$$

$$\langle \Phi_{\mathrm{cur}}(x_1) \ldots \Phi_{\mathrm{cur}}(x_n) \rangle$$

$$= \frac{\delta^n S_{\mathrm{eff}}}{\delta \Phi_{\mathrm{sh}}(x_1) \ldots \delta \Phi_{\mathrm{sh}}(x_n)}$$

correlation functions from AdS

correlation function from **CFT**

$\phi_{\mathbf{SYM}}$ fields of boundary conformal theory, e.g. SYM

$$S(\phi_{\mathbf{SYM}})$$

$$\Phi_{\mathsf{cur}} = \Phi_{\mathsf{cur}}(\phi_{\mathbf{SYM}})$$

$$\mathbf{V} = \int d^d x \; \mathbf{\Phi}_{\mathsf{sh}}(\mathbf{x}) \mathbf{\Phi}_{\mathsf{cur}}(\mathbf{x})$$

$$e^{-\mathbf{S_{cft}}} = \int D\phi_{\mathbf{SYM}} e^{-S(\phi_{\mathbf{SYM}})+\mathbf{V}}$$

AdS/CFT

$$s_{eff}(\Phi_{sh}) \stackrel{?}{=} s_{cft}(\Phi_{sh})$$

$S_{\text{eff}}(\Phi_{\text{sh}})$ for low spin fields

via AdS/CFT

	1998 – 1999
scalar field	**GKP, Witten**
massless spin-1	**Freedman et.al.**
massless spin-2	**Liu, Tseytlin**
massive spin-1	**Mueck, Viswanathan**
massive spin-2	**Polishchuk**

Goal

Find

$$S_{\text{eff}}(\Phi_{\text{sh}})$$

for **arbitrary spin** fields

by using AdS/CFT

scalar

$$S = \int d^d x dz \mathcal{L}$$

$$\mathcal{L} = \frac{1}{2}\sqrt{g}(g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi + m^2 \Phi^2)$$

$$\Phi = z^{\frac{d-1}{2}} \phi$$

scalar

$$\mathcal{L} = \frac{1}{2}|\partial^{\mathbf{a}}\phi|^{\mathbf{2}} + \frac{1}{2}|\mathcal{T}_\nu\phi|^{\mathbf{2}}$$

$$\mathcal{T}_\nu \equiv \partial_z + \frac{\nu}{z}$$

$$\nu = \sqrt{m^2 + \frac{d^2}{4}}$$

scalar

Solution to Dirichlet problem

$$\left(\Box + \partial_{\mathbf{z}}^{\mathbf{2}} - \frac{\nu^2}{z^2}\right)\phi = \mathbf{0}$$

$$\phi(\mathbf{x},\mathbf{z}) \quad \xrightarrow{z\longrightarrow 0} \quad \mathbf{z}^{-\nu+\frac{1}{2}}\ \phi_{\mathsf{sh}}(\mathbf{x})$$

$$\phi(\mathbf{x},\mathbf{z}) \quad \xrightarrow{z\longrightarrow \infty} \quad 0$$

scalar

Solution to Dirichlet problem

$$\phi(\mathbf{x}, \mathbf{z}) = \int d^d y \, \mathrm{G}_\nu(\mathbf{x} - \mathbf{y}, \mathbf{z}) \phi_{\mathsf{sh}}(\mathbf{y})$$

$$\mathrm{G}_\nu(\mathbf{x}, \mathbf{z}) = \frac{z^{\nu + \frac{1}{2}}}{(z^2 + |x|^2)^{\nu + \frac{d}{2}}}$$

scalar

$$S_{\mathsf{eff}} = \int d^d x \, \mathcal{L}_{\mathsf{eff}}|_{z \to 0}$$

$$\mathcal{L}_{\mathsf{eff}} = \phi \mathcal{T}_{\nu} \phi$$

scalar

Effective action

$$\mathbf{S}_{\mathsf{eff}} = \mathbf{c_0} \int d^d x_1 d^d x_2 \; \mathbf{\Gamma_{12}}$$

$$\mathbf{c_0} = \nu, \qquad \nu = \sqrt{m^2 + \frac{d^2}{4}}$$

$$\mathbf{\Gamma_{12}} = \frac{\phi_{\mathsf{sh}}(x_1)\phi_{\mathsf{sh}}(x_2)}{|x_{12}|^{2\nu+d}}$$

$$|x_{12}|^2 = (x_1 - x_2)^a (x_1 - x_2)^a$$

spin-1

$$\mathcal{L} = -\frac{1}{4} F^{AB} F^{AB}$$

$$F^{AB} = D^A \phi^B - D^B \phi^A$$

spin-1

bulk $\mathbf{so(d,1)} \to$ boundary $\mathbf{so(d-1,1)}$

$$\phi^{\mathbf{A}} \quad = \quad \phi^{\mathbf{a}} \oplus \phi^{\mathbf{z}}$$

$$\phi \equiv \phi^{z}$$

$$\phi^{+} \equiv \phi^{0} + \phi^{d-1}$$

Popular (**and important !**) gauge conditions

$\phi = 0$ radial gauge

$\phi^+ = 0$ light-cone gauge

way out

use differential gauge condition

Lorentz gauge ?

de Donder gauge ?

spin-1

standard Lorentz gauge

$$D^A \phi^A = 0$$

leads to

coupled equations

$$(\Box + \partial_z^2 - \frac{m_1^2}{z^2})\phi^{\mathbf{a}} + \partial^a \phi = 0$$

$$(\Box + \partial_z^2 - \frac{m_0^2}{z^2})\phi + \partial^a \phi^{\mathbf{a}} = 0$$

"Technical" problems with standard

Lorentz and de Donder gauge conditions

1) **Coupled equations**

2) For spin **2, 3, 4, ...**

solutions are expressible

in terms of **Heyn functions**

Little is known about **Heun functions**

asymptotic behavior ???

recurrent relations ???

spin-1

Modified Lorentz gauge

$$\mathbf{D^A}\phi^{\mathbf{A}} + \frac{2}{R}\phi = 0$$

RRM, 1999

Polchinski and Strassler 2001

gives

Decoupled equations

spin-1

Decoupled equations

$$(\Box + \partial_z^2 - \frac{{\nu_\mathbf{1}}^2}{z^2})\phi^\mathbf{a} = 0$$

$$(\Box + \partial_z^2 - \frac{{\nu_\mathbf{0}}^2}{z^2})\phi = 0$$

$$\nu_\mathbf{1} = \tfrac{d-2}{2}\,, \qquad \nu_\mathbf{0} = \tfrac{d-4}{2}$$

spin-1

Solution to Dirichlet problem

$$\phi^a(x,z) = \int d^d y \mathbf{G}_{\nu_1}(\mathbf{x}-\mathbf{y},\mathbf{z}) \ \phi^a_{\mathsf{sh}}(y)$$

$$\phi(x,z) = \int d^d y \mathbf{G}_{\nu_0}(\mathbf{x}-\mathbf{y},\mathbf{z}) \ \phi_{\mathsf{sh}}(y)$$

$$\mathrm{G}_\nu(\mathrm{x},\mathrm{z}) \equiv \frac{z^{\nu+1/2}}{(z^2+|x|^2)^{\nu+\frac{d}{2}}}$$

spin-1

$$S_{\mathsf{eff}} = \int d^d x\, \mathcal{L}_{\mathsf{eff}}|_{z\to 0}$$

$$\mathcal{L}_{\mathsf{eff}} = \phi^a \mathcal{T}_{\nu_{\mathbf{1}}} \phi^a + \phi \mathcal{T}_{\nu_{\mathbf{0}}} \phi$$

$$\mathcal{T}_\nu = \partial_z + \frac{\nu}{z}$$

spin-1

Effective action

$$S_{\text{eff}} = \int d^d x_1 d^d x_2 \boldsymbol{\Gamma_{12}}$$

$$\Gamma_{12} = \frac{\phi^a_{\text{sh}}(x_1)\phi^a_{\text{sh}}(x_2)}{|x_{12}|^{2(d-1)}}$$

$$+ \frac{\phi_{\text{sh}}(x_1)\phi_{\text{sh}}(x_2)}{|x_{12}|^{2(d-2)}}$$

Modified Lorentz gauge for bulk AdS fields

$$\mathbf{D^A \phi^A + 2\phi = 0}$$

has left-over gauge symmetry

$$\delta\phi^A = \partial^A \xi$$

$$(\Box + \partial_z^2 - \frac{{\nu_1}^2}{z^2})\xi = 0$$

$$\xi(\mathbf{x}, \mathbf{z}) = \int d^d y G_{\nu_1}(x - y, z) \ \xi^{\mathbf{a}}_{\mathsf{sh}}(\mathbf{y})$$

Modified Lorentz gauge for bulk AdS fields

$$\mathbf{D^A \phi^A + 2\phi = 0}$$

leads to

differential constraint for shadow fields

$$\boldsymbol{\partial^a \phi^a_{\mathsf{sh}} + \phi_{\mathsf{sh}} = 0}$$

Gauge symmetry of differential constraint

$$\delta\phi^a_{\mathsf{sh}} = \partial^a \xi_{\mathsf{sh}}$$

$$\delta\phi_{\mathsf{sh}} = -\Box\xi_{\mathsf{sh}}$$

$$\Gamma_{12} = \phi^a_{\mathsf{sh}}(x_1)\frac{\mathbf{O}^{\mathbf{ab}}_{\mathbf{12}}}{|x_{12}|^{2(d-1)}}\phi^b_{\mathsf{sh}}(x_2)$$

$$\mathbf{O}^{\mathbf{ab}}_{\mathbf{12}} = \eta^{\mathbf{ab}} - \mathbf{2}\frac{x^a_{12}x^b_{12}}{|x_{12}|^2}$$

Light-cone frame

$$x^a = x^+, x^-, x^i, \qquad i = 1, \ldots d-2$$

$$x^\pm = x^{d-1} \pm x^0$$

$$\phi^a = \phi^+, \phi^-, \phi^i$$

$\phi^+_{\mathsf{sh}} = 0$ light-cone gauge

Solution to differential constraint

$$\phi^-_{\mathsf{sh}} = -\frac{\partial^j}{\partial_-}\phi^j_{\mathsf{sh}} - \frac{1}{\partial_-}\phi_{\mathsf{sh}}$$

Light-cone gauge fixed S_{eff}

$$\mathbf{S}_{\text{eff}}^{\text{light}-\text{cone}} = \int d^d x_1 d^d x_2 \, \mathbf{\Gamma}_{\mathbf{12}}^{\text{light}-\text{cone}}$$

$$\Gamma_{12}^{\text{light}-\text{cone}} = \frac{\phi^i_{\text{sh}}(x_1)\phi^i_{\text{sh}}(x_2)}{|x_{12}|^{2(d-1)}}$$

$$+ \frac{\phi_{\text{sh}}(x_1)\phi_{\text{sh}}(x_2)}{|x_{12}|^{2(d-2)}}$$

ϕ^i_{sh}, ϕ_{sh} unconstrained fields

Bulk spin-1 - boundary conformal current

Solution for non-normalizable modes

$$\phi^a(x,z) = \mathbf{U}_{\nu_1}\ \phi^a_{\mathsf{cur}}(x)$$

$$\phi(x,z) = \mathbf{U}_{\nu_0}\ \phi_{\mathsf{cur}}(x)$$

$$\mathbf{U}_\nu \equiv \sqrt{z}\mathbf{J}_\nu(\mathbf{z}\sqrt{\Box})\Box^{-\nu/2}$$

$$\mathbf{U}_\nu \equiv \mathbf{z}^{\nu+\frac{1}{2}}(\mathbf{1}+\mathbf{z}^{\mathbf{2}}\Box+\ldots)$$

Bessel

modified Lorentz gauge for bulk fields

$$\mathbf{D^A}\phi^{\mathbf{A}} + \mathbf{2}\phi = \mathbf{0}$$

leads to differential constraint for boundary currents

$$\mathbf{D^A}\phi^{\mathbf{A}} + \mathbf{2}\phi = U_{\nu_1}(\partial^{\mathbf{a}}\phi^{\mathbf{a}}_{\mathsf{cur}} + \Box\phi_{\mathsf{cur}})$$

$$\partial^a\phi^a_{\mathsf{cur}} + \Box\phi_{\mathsf{cur}} = 0$$

Modified Lorentz gauge for bulk AdS fields

$$\mathbf{D^A \phi^A + 2\phi = 0}$$

has left-over gauge symmetry

$$\delta\phi^A(x,z) = \partial^A \xi(x,z)$$

$$(\Box + \partial_z^2 - \frac{\nu^2}{z^2})\xi = 0$$

$$\xi(x,z) = \mathbf{U}_\nu\ \xi_{\rm cur}(x)$$

$$\delta\phi^a_{\rm cur} = \partial^a \xi_{\rm cur}$$

$$\delta\phi_{\rm cur} = -\xi_{\rm cur}$$

$$\partial^{\mathbf{a}}\phi_{\text{cur}}^{\mathbf{a}} + \Box\phi_{\text{cur}} = \mathbf{0}$$

$$\delta\phi_{\text{cur}}^{a} = \partial^{a}\xi_{\text{cur}}$$

$$\delta\phi_{\text{cur}} = -\xi_{\text{cur}}$$

1) Stueckelberg gauge condition $\phi_{\text{cur}} = 0$

$$\partial^{\mathbf{a}}\phi_{\text{cur}}^{\mathbf{a}} = \mathbf{0}$$

2) Light-cone gauge condition $\phi^{+} = 0$

$$\phi_{\text{cur}}^{i}\,, \qquad \phi_{\text{cur}}$$

$$\phi_{\text{cur}}^{-} = -\frac{\partial^{i}}{\partial_{-}}\phi_{\text{cur}}^{i} - \frac{\Box}{\partial_{-}}\phi_{\text{cur}}$$

Intermediate Conclusions

1) Use of Modified Lorentz gauge leads to generalized **gauge invariant formulation of CFT**

2) Standard formulation of CFT and light-cone gauge CFT are obtained by using Stueckelberg gauge fixing and light-cone gauge

Spin-2

Einstein equation for h^{AB}

$$D^2 h^{AB} + \ldots = 0$$

Standard de Donder gauge

$$\mathbf{D^B h^{AB}} - \frac{1}{2}\mathbf{D^A h} = 0$$

leads to **coupled EOM**

Spin-2

modified de Donder gauge

RRM, 2008

$$\mathbf{D^B h^{AB}} - \frac{1}{2}\mathbf{D^A h} + \mathbf{2h^{zA}} - \eta^{\mathbf{zA}}\mathbf{h} = 0$$

leads to **decoupled** equations

$$\mathbf{so(d,1)} \quad \Longrightarrow \quad \mathbf{so(d-1,1)}$$

$$h^{AB} \quad = \quad h^{ab} \oplus h^{za} \oplus h^{zz}$$

$$\phi^{\mathbf{ab}} \equiv h^{ab} + \eta^{ab} h^{zz}\,, \qquad \phi^{\mathbf{a}} \equiv h^{za}\,, \qquad \phi \equiv h^{zz}$$

spin-2: Decoupled equations

$$(\Box + \partial_z^2 - \frac{{\nu_{\mathbf{2}}}^2}{z^2})\phi^{\mathbf{ab}} = 0$$

$$(\Box + \partial_z^2 - \frac{{\nu_{\mathbf{1}}}^2}{z^2})\phi^{\mathbf{a}} = 0$$

$$(\Box + \partial_z^2 - \frac{{\nu_{\mathbf{0}}}^2}{z^2})\phi = 0$$

$$\nu_{\mathbf{2}} = \frac{d}{2}, \qquad \nu_{\mathbf{1}} = \frac{d-2}{2}, \qquad \nu_{\mathbf{0}} = \frac{d-4}{2}$$

Spin-2

Solution to Dirichlet problem

$$\phi^{ab}(x,z) = \int d^d y \mathbf{G}_{\nu_2}(\mathbf{x}-\mathbf{y},\mathbf{z})\ \phi^{ab}_{\mathsf{sh}}(y)$$

$$\phi^{a}(x,z) = \int d^d y \mathbf{G}_{\nu_1}(\mathbf{x}-\mathbf{y},\mathbf{z})\ \phi^{a}_{\mathsf{sh}}(y)$$

$$\phi(x,z) = \int d^d y \mathbf{G}_{\nu_0}(\mathbf{x}-\mathbf{y},\mathbf{z})\ \phi_{\mathsf{sh}}(y)$$

$$\mathbf{G}_\nu(\mathbf{x},\mathbf{z}) \equiv \frac{z^{\nu+1/2}}{(z^2+|x|^2)^{\nu+\frac{d}{2}}}$$

Spin-2. Effective action

$$\mathbf{S}_{\mathsf{eff}} = \int d^d x \, \mathcal{L}_{\mathsf{eff}}|_{\mathbf{z}\to\mathbf{0}}$$

$$\mathcal{L}_{\mathsf{eff}} = \phi^{\mathbf{ab}} \mathcal{T}_{\nu_{\mathbf{2}}} \phi^{\mathbf{ab}} + \phi^{\mathbf{a}} \mathcal{T}_{\nu_{\mathbf{1}}} \phi^{\mathbf{a}} + \phi \mathcal{T}_{\nu_{\mathbf{0}}} \phi$$

$$\mathcal{T}_\nu = \partial_z + \frac{\nu}{z}$$

$$\nu_2 = \frac{d}{2}, \qquad \nu_1 = \frac{d-2}{2}, \qquad \nu_0 = \frac{d-4}{2}$$

Spin-2

Effective action

$$\mathrm{S}_{\mathrm{eff}} = \int d^d x_1 d^d x_2 \; \mathbf{\Gamma_{12}}$$

$$\Gamma_{12} = \frac{\phi^{ab}_{\mathrm{sh}}(x_1)\phi^{ab}_{\mathrm{sh}}(x_2)}{|x_{12}|^{2d}}$$

$$+ \; \frac{\phi^{a}_{\mathrm{sh}}(x_1)\phi^{a}_{\mathrm{sh}}(x_2)}{|x_{12}|^{2(d-1)}}$$

$$+ \; \frac{\phi_{\mathrm{sh}}(x_1)\phi_{\mathrm{sh}}(x_2)}{|x_{12}|^{2(d-2)}}$$

Modified de Donder gauge for bulk AdS fields

$$\mathbf{D^B h^{AB}} - \frac{1}{2}\mathbf{D^A h} + \mathbf{2h^{zA}} - \eta^{\mathbf{zA}}\mathbf{h} = 0$$

leads to

differential constraints for shadow fields

$$\partial^b \phi_{\mathrm{sh}}^{ab} + \partial^a \phi_{\mathrm{sh}}^{bb} + \phi_{\mathrm{sh}}^{a} = 0$$

$$\partial^a \phi_{\mathrm{sh}}^{a} + \Box \phi_{\mathrm{sh}}^{aa} + \phi_{\mathrm{sh}} = 0$$

ϕ_{sh}^{aa} can be gauged away

$$\partial^b \phi_{\mathrm{sh}}^{ab} + \phi_{\mathrm{sh}}^{a} = 0$$

$$\partial^a \phi_{\mathrm{sh}}^{a} + \phi_{\mathrm{sh}} = 0$$

On-shell left-over gauge symmetries of bulk AdS fields lead to gauge symmetries of shadow fields

$$\delta\phi_{\text{sh}}^{ab} = \partial^a\xi_{\text{sh}}^b + \partial^b\xi_{\text{sh}}^a + \eta^{ab}\xi_{\text{sh}}$$

$$\delta\phi_{\text{sh}}^{a} = \partial^a\xi_{\text{sh}} + \Box\xi_{\text{sh}}^a$$

$$\delta\phi_{\text{sh}} = \Box\xi_{\text{sh}}$$

ϕ_{sh}^{aa} is Stueckelberg field

ϕ_{sh}^{a}, ϕ_{sh} are not Stueckelberg fields

Arbitrary spin-s AdS field

$$\Phi^{A_1 \ldots A_s}$$

Fronsdal action for free fields

Vasiliev theory of interacting fields

Impose **modified** **de Donder gauge**

$$D^A\Phi^{AA_2\ldots A_s} - \frac{1}{2}D^{A_2}\Phi^{AAA_3\ldots A_s}$$

$$+2\Phi^{zA_2\ldots A_s} - \eta^{zA_2}\Phi^{AAA_3\ldots A_s} = 0$$

Decompose

$$\mathrm{so}(d,1) \longrightarrow \mathrm{so}(d-1,1)$$

$$\Phi^{A_1\ldots A_s} = \Phi^{a_1\ldots a_s}$$

$$\Phi^{a_1\ldots a_{s-1}}$$

$$\ldots\ldots$$

$$\Phi^{a_1 a_2}$$

$$\Phi^{a_1}$$

$$\Phi$$

$$\phi^{\mathbf{a_1 \dots a_s}} = \mathbf{\Phi}^{\mathbf{a_1 \dots a_s}} + \dots$$

$$\phi^{\mathbf{a_1 \dots a_{s-1}}} = \mathbf{\Phi}^{\mathbf{a_1 \dots a_{s-1}}} + \dots$$

$$\dots\dots$$

$$\phi^{\mathbf{a_1 a_2}} = \mathbf{\Phi}^{\mathbf{a_1 a_2}} + \dots$$

$$\phi^{\mathbf{a}} = \mathbf{\Phi}^{\mathbf{a_1}}$$

$$\phi = \mathbf{\Phi}$$

Decoupled equations

$$(\Box + \partial_z^2 - \frac{{\nu_{\mathbf{s'}}}^{\mathbf{2}}}{z^2})\phi^{\mathbf{a_1 \dots a_{s'}}} = 0$$

$$\nu_{s'} = s' + \frac{d-4}{2}$$

$$\phi^{\mathbf{a_1 \dots a_{s'}}}(\mathbf{x}, \mathbf{z}) = \int d^d y G_{\nu_{\mathbf{s'}}}(x-y, z) \phi_{\mathsf{sh}}^{\mathbf{a_1 \dots a_{s'}}}(\mathbf{y})$$

$$S_{\text{eff}} = \int dx_1^d dx_2^d \boldsymbol{\Gamma}_{\mathbf{12}}$$

$$\boldsymbol{\Gamma}_{\mathbf{12}} = \frac{\phi_{\text{sh}}^{\mathbf{a_1 \ldots a_s}} \phi_{\text{sh}}^{\mathbf{a_1 \ldots a_s}}}{|x_{12}|^{2(s+d-2)}}$$

$$+ \quad \frac{\phi_{\text{sh}}^{\mathbf{a_1 \ldots a_{s-1}}} \phi_{\text{sh}}^{\mathbf{a_1 \ldots a_{s-1}}}}{|x_{12}|^{2(s+d-3)}}$$

$$+ \quad \ldots\ldots\ldots\ldots$$

$$+ \quad \ldots\ldots\ldots\ldots\ldots$$

$$+ \quad \frac{\phi_{\text{sh}}^{\mathbf{a}} \phi_{\text{sh}}^{\mathbf{a}}}{|x_{12}|^{2(d-1)}}$$

$$+ \quad \frac{\phi_{\text{sh}} \phi_{\text{sh}}}{|x_{12}|^{2(d-2)}}$$

Use of differential constraints for shadow fields leads to

$$\Gamma_{12} = \phi_{\mathsf{sh}}^{a_1\ldots a_s}(x_1)\frac{O_{12}^{a_1b_1}\ldots O_{12}^{a_sb_s}}{|x_{12}|^{2(s+d-2)}}\phi_{\mathsf{sh}}^{b_1\ldots b_s}(x_2)$$

$$\mathbf{O_{12}^{ab}} = \eta^{\mathbf{ab}} - \mathbf{2}\frac{x_{12}^a x_{12}^b}{|x_{12}|^2}$$

Massless: Normalization factor

$$S_{\text{eff}} = \mathbf{c(s,d)} \int dx_1^d dx_2^d \Gamma_{12}$$

RRM, 2009

$$\mathbf{c(s,d)} = \frac{(2s+d-3)(2s+d-4)}{2s!(s+d-3)}$$

$$\mathbf{c(1,d)} = \frac{1}{2}(\mathbf{d-2}) \qquad \text{Freedman et.al.}$$

$$\mathbf{c(2,d)} = \frac{d(d+1)}{4(d-1)} \qquad \text{Liu, Tseytlin}$$

Generalization to **massive fields** is straightforward

use gauge invariant formulation with **Stueckelberg fields**

$$\mathcal{L} = -\frac{1}{4}F^{AB}F^{AB} - \frac{1}{2}(\partial^A\varphi + m\phi^A)^2$$

$F^{AB} = D^A\phi^B - D^B\phi^A$

$$\delta\phi^A = \partial^A\xi$$

$$\delta\varphi = -m\xi$$

$$\mathbf{D^A}\phi^{\mathbf{A}} + \mathbf{m}\varphi + \mathbf{2}\phi^{\mathbf{z}} = \mathbf{0}$$

Massive: Normalization factor

$$S_{\mathsf{eff}} = \mathbf{c}(\mathbf{m}, \mathbf{s}, \mathbf{d}) \int dx_1^d dx_2^d \Gamma_{12}$$

$$\mathbf{c}(\mathbf{m}, \mathbf{s}, \mathbf{d}) = \frac{\kappa(2\kappa + 2\mathbf{s} + \mathbf{d} - 2)}{\mathbf{s}!(2\kappa + \mathbf{d} - 2)}$$

$$\kappa \equiv \sqrt{\mathbf{m}^2 + (\mathbf{s} + \frac{\mathbf{d} - 4}{2})^2}$$

RRM, 2011

Spin-2 conformal current (energy-momentum tensor) Standard approach

$$\mathbf{T^{ab}} \qquad - \, spin \ 2 \ conformal \quad current$$

$$\partial^{\mathbf{a}}\mathbf{T^{ab}} = \mathbf{0}$$

$$\mathbf{T^{aa}} = \mathbf{0}$$

Conformal dimension

$$\mathbf{\Delta = d}$$

Spin-2 current. Gauge inv. approach

Fields	$Conf.dim$
$\phi_{\rm cur}^{\mathbf{ab}}$	d
$\phi_{\rm cur}^{\mathbf{a}}$	$d-1$
$\phi_{\rm cur}$	$d-2$

Spin 2. Currents. Differential constraints

$$\partial^b \phi^{ab}_{\text{cur}} + \partial^a \phi^{bb}_{\text{cur}} + \Box \phi^a_{\text{cur}} = 0$$

$$\partial^a \phi^a_{\text{cur}} + \phi^{aa}_{\text{cur}} + \Box \phi_{\text{cur}} = 0$$

ϕ^a_{cur} ϕ_{cur} can be gauged away

$$\partial^b \phi^{ab}_{\text{cur}} = 0$$

$$\phi^{aa}_{\text{cur}} = 0$$

Spin 2. Currents. Gauge transformations

$$\delta\phi^{ab}_{\mathsf{cur}} = \partial^a\xi^b_{\mathsf{cur}} + \partial^b\xi^a_{\mathsf{cur}} + \eta^{ab}\Box\xi_{\mathsf{cur}}$$

$$\delta\phi^{a}_{\mathsf{cur}} = \partial^a\xi_{\mathsf{cur}} + \xi^a_{\mathsf{cur}}$$

$$\delta\phi_{\mathsf{cur}} = \xi_{\mathsf{cur}}$$

ϕ^a_{cur}, ϕ_{cur} Stueckelberg fields

Energy-momentum tensor in gauge inv.app.

$$T^{ab} = \phi^{ab}_{\text{cur}} + \partial^a \phi^b_{\text{cur}} + \partial^b \phi^a_{\text{cur}}$$

$$+ \partial^a \partial^b \phi_{\text{cur}} + \eta^{ab} \Box \phi_{\text{cur}}$$

1) T^{ab} is gauge invariant

2) $\partial^a T^{ab} = 0\,,$ $\qquad T^{aa} = 0$

amount to differential constraints

for ϕ^{ab}_{cur}, ϕ^a_{cur}, ϕ_{cur}.

summary of our study of AdS/CFT

1) Bulk fields are taken in **modified de-Donder gauge**

2) **Modified de Donder gauge** leads to **decoupled equations** of motion with on-shell leftover gauge symmetries

3) leftover **gauge symmetries of bulk fields** correspond to **gauge symmetries of boundary currents and shadow fields**,

4) **Modified de Donder gauge** for bulk fields corresponds to **differential constraints** for boundary conformal currents and shadows

5) normalizable solutions $\longrightarrow$ currents

non-normalizable solutions $\longrightarrow$ shadows

Our currents and shadows

correspond to

bulk AdS fields taken

in modified de Donder gauge

Work for future

Generalization of modified de Donder gauge to

non-conformal geometries

$$ds^2 = \frac{e^{f(z)}}{z^2}(dx^a dx^a + dzdz)$$

$\mathbf{f = 0}$ for AdS

$\mathbf{f(z) \neq 0}$ non-conformal