

CFT limit of $SU(N_c)$ perturbative expressions for the sum rules
of the polarized DIS

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Plan of Presentation

Introduction – Reminding of the concept of Conformal Symmetry

Existing perturbative Quenched U(1) identity $O(E^3)$ result: Coefficient functions of Ellis-Jaffe and Bjorken sum rules coincides! **Q1** Accident or Not ?

A1

- Not! **Statement: true in all orders of PT**

Explanation due to Kataev- 2010 derivation of new Crewther-type relation from triangle diagram of Axial - V-V currents for the Green function using first OPE expansion of VV -currents, which define EJ sum rule **SI** in the product with Axial current and comparing with basic **non-singlet** Crewther relation for non-singlet A-V-V triangle Crewther 1972

Positive conclusion of discussion between Kataev 1996 and Crewther 1997

A2

The conformal-invariant result for **DIS** sum rules at $O(A_s^3)$ -level the case of $SU(N_c)$ Kataev and Mikhailov 2010, Teor.Mat.Fiz. 2012

Conclusions

- 1) Possible applications- tests of future higher-order EJ SR analytical calculations- may be of interest for JLAB, HERA, studies
- 2) Other Comments, including theoretical ones

Conformal Invariance

is valid in the quark-parton model limit, and perturbative quenched QED. It is the symmetry under the following transformations of coordinates :

1. Translations $x'^{\mu} = x^{\mu} + \alpha^{\mu}$ with 4 parameters α^{μ} ,
2. Scale (or dilaton) transformation $x'^{\mu} = \rho x^{\mu}$ with 1 parameter $\rho > 0$,
3. Special conformal transformations $x'^{\mu} = \frac{x^{\mu} + \beta^{\mu} x^2}{1 + 2\beta x + \beta^2 x^2}$ with 4 parameters β^{μ} and
4. Homogeneous Lorentz transformations $x'^{\mu} = \Lambda_{\nu}^{\mu} x^{\nu}$, which also contain 4 parameters.
5. Consequences are widely studied at present, though in renormalized QFT models the CI is violated- appearance of β -function and the effects of running of the coupling constants- QCD, QED

Bjorken polarized sum rule:

$$B_{jp}(Q^2) = \int_0^1 (g_1^{lp}(x, Q^2) - g_1^{ln}(x, Q^2)) dx = \frac{1}{6} g_a C_{B_{jp}}^{ns}(A_s) \quad (1)$$

Depends from Q^2 through the running of $A_s(Q^2) = \alpha_s(Q^2)/(4\pi)$,
 Consider pqQED limit- $C_F = 1$ $N_F = 0$, $A = \alpha/(4\pi)$

$$C_{B_{jp}}^{ns} = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3 - \left(\frac{4823}{8} + 96\zeta(3) \right) A^4 + O(A^5) \quad (2)$$

$O(A^3)$ Larin and Vermaseren (1991); $O(A^4)$ - Baikov, Chetyrkin, Kuhn (2010)

Ellis-Jaffe sum rule :

$$EJ^{p(n)}(Q^2) = \int_0^1 g_1^{lp(n)}(x, Q^2) dx = C_{B_{jp}}^{ns}(A_s(Q^2)) \left(\pm \frac{1}{12} a_3 + \frac{1}{36} a_8 \right) + C_{EJ_p}^s(Q^2) \frac{1}{9} \Delta\Sigma(Q^2) \quad (3)$$

where $a_3 = \Delta u - \Delta d$, $a_8 = \Delta u + \Delta d - 2\Delta s$, Δu , Δd and Δs are the polarized distributions and $\Delta\Sigma$ depends from the scheme choice. In the \overline{MS} -scheme it is defined as $\Delta\Sigma = \Delta u + \Delta d + \Delta s$.

Note, that in the \overline{MS} -scheme the definition of the singlet coefficient function reads (Larin (1994), Larin, van Ritbergen, Vermaseren (1997)).

$$C_{EJp}^s = \overline{C}_{EJp}^s / Z_5^s \quad .$$

In the perturbative quenched QED the result is

$$\overline{C}_{EJ}^s = 1 - 7A + \frac{89}{2}A^2 - \left(\frac{1397}{6} - 96\zeta(3) \right) A^3 + O(A^4) \quad . \quad (4)$$

Z_5^s is the finite renormalization constants of $\overline{\Psi}\gamma_\mu\gamma_5\Psi$ current, Z_5^{ns} is the renormalization constant of $\overline{\Psi}\gamma_5(\lambda^a/2)\Psi$ -current . They were evaluated in Larin (1994) and Larin and Vermaseren (1991). In the limit of pqQED we have

$$Z_5^s(pqQED) = Z_5^{ns}(pqQED) = 1 - 4A + 22A^2 + \left(-\frac{370}{3} + 96\zeta_3 \right) A^3 + O(A^4) \quad . \quad (5)$$

Nontrivial **s**cheme-independent pqQED consequence of QCD results for SI and NS coefficient functions of EJ sum rule Kataev (2010):

$$C_{EJp}^s(A) = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3 + O(A^4) = C_{Bjp}^{ns}(A) \quad (6)$$

- Q1: What is the theoretical explanation ? Q2: Is this result true in all orders of PT ?
 Q3 : Is there any Q^2 -dependence ?

A1: Follow from Crewther-type relations for AVV diagrams in CI limit

A2: Valid in all orders of PT A3: No Q^2 dependence- fixed coupling constant

Proof of A1: Using OPE for

$$T_{\mu\alpha\beta}^{ab}(p, q) = i \int \langle 0 | TA_{\mu}(y) V_{\alpha}^a(x) V_{\beta}^b(0) | 0 \rangle e^{ipx+iqy} dx dy = \delta^{ab} \Delta_{\mu\alpha\beta}^{(1-loop)}(p, q) \quad (7)$$

where $A_{\mu} = \bar{\psi} \gamma_{\mu} \gamma_5 \psi$ and

$$T_{\mu\alpha\beta}^{abc}(p, q) = i \int \langle 0 | TA_{\mu}^a(y) V_{\alpha}^b(x) V_{\beta}^c(0) | 0 \rangle e^{ipx+iqy} dx dy = d^{abc} \Delta_{\mu\alpha\beta}^{(1-loop)}(p, q) \quad (8)$$

The consideration of the first and second triangle graphs+ the concept of CI give the relations, derived in p -space [Kataev 1996](#) for details [Gabdadze and Kataev 1995](#) and in x -space by [Crewther 1972](#)

$$C_{EJp}^s(A) \times C_D^A(A) = 1 \quad (1996) \quad C_{Bjp}^{ns}(A) \times C_D^{ns}(A) = 1 \quad (1972) \quad (9)$$

where $C_D^{A(ns)}$ are defined from taking $q^2 \frac{d}{dq^2}$ of

$$i \int \langle 0 | TA_{\mu}(x) A_{\nu}(0) | 0 \rangle e^{iqx} dx = \Pi_{\mu\nu}^{ax}(q^2) \quad (10)$$

$$i \int \langle 0 | TA_{\mu}^a(x) A_{\nu}^b(0) | 0 \rangle e^{iqx} dx = \delta^{ab} \Pi_{\mu\nu}^{ns}(q^2) . \quad (11)$$

In the massless limit chiral symmetry is exact, thus

$$C_D^A(A) = C_D^{ns}(A)$$

Thus in the CI limit in all orders of PT

$$C_{EJp}^s(A) = C_{Bjp}^{ns}(A) \quad \text{Kataev 2010} \quad (12)$$

In pqQED with $A = \alpha/(4\pi)$ CI limit we have

$$C_{EJp}^s(A) = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3 = C_{Bjp}^{ns}(A) \quad (13)$$

In the case of the $SU(N_c)$ with $A_s = \alpha_s/(4\pi)$

$$\begin{aligned} C_{Bjp}^{ns} &= 1 - 3C_F A_s + \left(\frac{21}{2}C_F^2 - C_F C_A\right) A_s^2 \\ &+ \left[\left(-\frac{3}{2}C_F^3 - 65C_F^2 C_A - \left(\frac{523}{12} - 216\zeta_3\right)C_F C_A^2\right) A_s^3 + O(A_s^4) \right] = C_{EJ}^s(A_s) \end{aligned} \quad (14)$$

Numbers obtained from Crewther relation in Kataev and Mikhailov (2010-2012)

This is the CI predictions for $O(A_s^3)$ -approximation of $C_{EJp}^s(A_s)$

Conclusion

In the CI limit in $SU(N_c)$

1. It is possible to try to get $O(A_s^4)$ for C_{Bjp} - from [Kataev, Mikhailov \(2012\)](#)- strong test for analytical evaluation of Ellis-Jaffe sum rule at A_s^4 ($SU(N_c)$ results may be obtained soon).

2. In this CI limit we get the recover quark-parton model expressions for the ratios of polarized sum rules

$$\frac{EJ^{p(n)}}{Bjp} = \pm \frac{1}{2} + \frac{a_8}{6 a_3} + \frac{2\Delta\Sigma}{3 a_3} \quad (15)$$

where $a_3 = \Delta u - \Delta d$, $a_8 = \Delta u + \Delta d - 2\Delta s$, and $\Delta\Sigma = \Delta u + \Delta d + \Delta s$, $a_3 = g_A$, $a_8 = 3a_3 - 4D$

$$\frac{EJ^p}{Bjp} = 1 + \frac{2}{3} \frac{\Delta\Sigma - D}{a_3} \quad \frac{EJ^n}{Bjp} = \frac{2}{3} \frac{\Delta\Sigma - D}{a_3} \quad \frac{EJ^p}{Bjp} - \frac{EJ^n}{Bjp} = 1 \quad (16)$$

3. How to use CFT limit for the Bjorken sum rule in phenomenology ?

Use "Principle of Maximal Conformality" ([Brodsky and Wu 20111, 2012](#))

PMC= Generalization of BLM approach to higher level using

[Grunberg and Kataev 1991](#)+ β -expansion approach [Mikhailov, Quarks 2004- JHEP 2007](#)

$$\begin{aligned} C_{Bjp}^{ns} &= 1 - 3C_F A_s(Q^{PMC}) + \left(\frac{21}{2} C_F^2 - C_F C_A\right) A_s(Q^{PMC})^2 \\ &+ \left[\left(-\frac{3}{2} C_F^3 - 65 C_F^2 C_A - \left(\frac{523}{12} - 216\zeta_3\right) C_F C_A^2\right) A_s(Q^{PMC})^3 + O(A_s^4) \right] \end{aligned} \quad (17)$$