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# SOFT WALL MODEL WITH NEGATIVE DILATON AS A MODEL FOR THE AXIAL MESONS

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# Bottom-up AdS/QCD models

**Typical ansatz:**

$$S = \int d^4x dz \sqrt{g}\, F(z) \mathcal{L}$$

$$ds^2 = \frac{R^2}{z^2}(dx_\mu dx^\mu - dz^2), \qquad z > 0$$

Vector mesons:

$$z = \epsilon \to 0 \qquad V_M(x,\epsilon) \leftarrow \bar{q}\gamma_\mu q \text{ or } V_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu\gamma_5 q$$

$$m_5^2 R^2 = (\Delta - J)(\Delta + J - 4) \qquad J = 0, 1$$

**Precise sense of holography:** $W_{4D}[\varphi_0(x)] = S_{5D}[\varphi(x,\epsilon)]$

generating functional — effective action

The output of the holographic models: **Correlators**

Mass spectrum: Poles of the two-point correlator

Alternative way: find the mass spectrum is to solve e.o.m. $\varphi(x,z) = e^{ipx}\varphi(z)$

# Chiral symmetry breaking

The hard-wall model (Erlich et al., PRL 95, 261602 (2005))

$$S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$$

$$D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu},\ A_{L,R} = A^a_{L,R}t^a,\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

For $N_f = 2$ $\quad t^a = \sigma^a/2$

$V = (A_L + A_R)/2 \qquad A = (A_L - A_R)/2$

At $z = z_m$ one imposes certain gauge invariant boundary conditions on the fields.

**Let us try to “discover” CSB from a 5D approach (at least some igredients of) instead of inserting it by hand**

**Following the spirit of the holographic correspondence, we should do it on the level of a <u>free 5D action</u>**

A. Karch, E. Katz, D. T. Son, M. A. Stephanov, PRD 74, 015005 (2006)

**Soft-wall model:**

$$S = -\frac{c^2}{4}\int d^4x\, dz \sqrt{g}\, e^{-az^2} F_{MN}F^{MN}$$

$$F_{MN} = \partial_M V_N - \partial_N V_M,\ M = 0, 1, 2, 3, 4$$

Here **a > 0**. We will consider **a < 0.**

$$W_{4D}[\varphi_0(x)] = S_{5D}[\varphi(x,\epsilon)]$$

4D Fourier transform → source →

$$V^\mu(q,z) = v(q,z)V_0^\mu(q) \qquad v(q,\epsilon) = 1$$

E.O.M.:

$$\partial_z\left(\frac{e^{-az^2}}{z}\partial_z v\right) + \frac{e^{-az^2}}{z}q^2 v = 0$$

zero-mode:

$$v_0 = C_1 e^{az^2} + C_2$$

$$S = \frac{c^2}{2}\int d^4x V_0^\mu V_{0\mu}\, \frac{e^{-az^2}}{z}\, v\partial_z v\Big|_{z=\epsilon}^{z=\infty}$$

$$\int d^4x e^{iqx} \langle J_\mu(x) J_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_V(-q^2)$$

$$\Pi_V(-q^2) = c^2 \left. \frac{\partial_z v}{q^2 z} \right|_{z=\epsilon}$$

$$v(q,z) = \Gamma\left(1 - \frac{q^2}{4|a|}\right) e^{(a-|a|)z^2/2} U\left(\frac{-q^2}{4|a|}, 0; |a| z^2\right)$$

$$\Pi_V(-q^2) = c^2 \left[\frac{a-|a|}{q^2} - \frac{1}{2}\psi\left(1 - \frac{q^2}{4|a|}\right)\right] + \text{const}$$

$$\Pi_V(-q^2) = c^2 \left[\frac{a-|a|}{q^2} + \sum_{n=0}^{\infty} \frac{2|a|}{4|a|(n+1) - q^2}\right] + \text{const}$$

Scalar case: $m_n^2 = 2|a|\left(2n + \Delta - 1 + \frac{a}{|a|}\right)$

# The scalar sector

We had the pole in the vector correlator for ***a < 0***

$$\Pi_V(-q^2) = c^2 \left[ \frac{a - |a|}{q^2} + \sum_{n=0}^{\infty} \frac{2|a|}{4|a|(n+1) - q^2} \right] + \text{const}$$

It cannot be a physical massless vector state because substituting

$$A_\mu(q,z) = \sum_{n=0}^{\infty} A_\mu^{(n)}(q) v_n(z)$$

back into the action we will obtain

$$S = c^2 \int d^4x \sum_{n=0}^{\infty} \left[ -\frac{1}{4} \left(F_{\mu\nu}^{(n)}\right)^2 + m_n^2 \left(A_\mu^{(n)}\right)^2 \right]$$

Consider $S = \int d^4x\, dz \sqrt{g}\, e^{-az^2} (\partial_M \Phi \partial^M \Phi - m^2 \Phi^2)$ $\qquad m^2 R^2 = \Delta(\Delta - 4)$

$$m_n^2 = 2|a| \left( 2n + \Delta - 1 + \frac{a}{|a|} \right), \qquad n = 0, 1, 2, \ldots \qquad \Longrightarrow \qquad \boxed{m_n^2 = 4|a|n}$$

for $a < 0 \quad \Delta = 2$

This corresponds to $m_5^2 R^2 = -4$ - the lowest possible mass in the AdS

Comparing with the vector spectrum we deduce $m_{\pi'} = m_{a_1}$

Particle Data (in MeV): $m_{\pi'} = 1300 \pm 100 \qquad m_{a_1} = 1230 \pm 40$

$$\Pi_V(Q^2)_{Q^2\to\infty} = \frac{c^2}{2}\left[\log\left(\frac{4|a|}{Q^2}\right) - \frac{2a}{Q^2} + \frac{4a^2}{3Q^4} + \mathcal{O}\left(\frac{a^4}{Q^8}\right)\right] \qquad Q^2 = -q^2$$

$$\Pi_V(Q^2)_{\mathrm{OPE}} = \frac{N_c}{24\pi^2}\log\left(\frac{\mu^2}{Q^2}\right) + \frac{\alpha_s}{24\pi}\frac{\langle G^2\rangle}{Q^4} + \xi\frac{\langle\bar{q}q\rangle^2}{Q^6} + \mathcal{O}\left(\frac{\mu^8}{Q^8}\right)$$

$$\Longrightarrow \qquad c^2 = \frac{N_c}{12\pi^2}$$

**QCD sum rules:**

$$\Pi_V(Q^2) = \sum_{n=0}^{\infty}\frac{2f_\pi^2}{Q^2+\Lambda(n+1/2)} + \mathrm{const}$$

$$\Pi_A(Q^2) = \frac{f_\pi^2}{Q^2} + \sum_{n=0}^{\infty}\frac{2f_\pi^2}{Q^2+\Lambda(n+1)} + \mathrm{const}$$

$$\Lambda = \frac{48\pi^2}{N_c}f_\pi^2$$

## No-wall model:

(S.S. Afonin, IJMPA 25, 3615 (2011))

$$S = -\frac{c^2}{4}\int d^4x\, dz\sqrt{g}\, e^{-az^2} F_{MN}F^{MN} \qquad V_M = e^{az^2/2}\tilde{V}_M$$

$$S = -\frac{1}{4}\int d^4x\, dz\sqrt{g}\left(\tilde{F}_{MN}\tilde{F}^{MN} + \frac{a^2z^4}{2}\tilde{V}_\mu\tilde{V}^\mu\right) + \frac{a}{2}\int d^4x\, \tilde{V}_\mu^2\Big|_{z=0}^{z=\infty}$$

$$v(z) \sim z^k e^{(a-|a|)z^2/2}, \qquad k > 0$$

$$S = \int d^4x\, dz\sqrt{g}\left(|D_M\varphi|^2 - m_\varphi^2\varphi^2 - \frac{1}{4}F_{MN}F^{MN}\right)$$

$$D_M = \partial_M - i\lambda V_M \qquad -\partial_z\left(\frac{\partial_z\varphi}{z^3}\right) + \frac{m_\varphi^2\varphi}{z^5} = 0$$

$$\varphi_0 \sim z^2 \quad \Longrightarrow \quad m_\varphi^2 = -4$$

$$\Longrightarrow \quad \Delta = 2$$

According to the holographic prescriptions: $\Phi(x,z)_{z\to 0} = z^{4-\Delta}\Phi_0(x) + z^\Delta \frac{\langle O(x)\rangle}{2\Delta - 4}$

Vector correlator of no-wall model:

$$\Pi_V(-q^2) = c^2 \left[ -\frac{|a|}{q^2} + \sum_{n=0}^{\infty} \frac{2|a|}{4|a|(n+1) - q^2} \right] + \text{const}$$

$$\Pi_V(Q^2)_{Q^2 \to \infty} = \frac{c^2}{2} \left[ \log\left(\frac{4|a|}{Q^2}\right) + \frac{4a^2}{3Q^4} + \mathcal{O}\left(\frac{a^4}{Q^8}\right) \right] \qquad Q^2 = -q^2$$

The no-wall model describes the discrete spectrum of axial mesons only!

$$A_L^M(x,\epsilon) \leftrightarrow \bar{q}_L\gamma^\mu q_L \text{ and } A_R^M(x,\epsilon) \leftrightarrow \bar{q}_R\gamma^\mu q_R$$

$$V = A_L + A_R,\ A = A_L - A_R$$

$$S = \int d^4x\, dz \sqrt{g} \left( |D_M\varphi|^2 + 4\varphi^2 - \frac{1}{4}F_L^2 - \frac{1}{4}F_R^2 \right)$$

$$D_M = \partial_M - i\lambda(A_{L,M} - A_{R,M})$$

$$S = \int d^4x\, dz \sqrt{g} \left( |\partial_M\varphi - i\lambda A_M\varphi|^2 + 4\varphi^2 - \frac{1}{8}F_A^2 - \frac{1}{8}F_V^2 \right)$$

## Phenomenology

$$m_n^2 = m_0^2(n+1), \qquad n = 0, 1, 2, \ldots.$$

$$m_n^2 = m_0^2\{1, 2, 3, 4, \ldots\}$$

**Particle Data:**

$$m_{\rho,n}^2 = m_\rho^2\{1, 3.6, 4.9, 6.0^{[?]}, 6.7^{[??]}, 7.7^{[?]}, 8.5^{[??]}\}$$

$$m_{a_1,n}^2 = m_{a_1}^2\{1, 1.8^{[?]}, 2.5^{[??]}, 3.4^{[??]}\}$$

SW model with UV cutoff:

(S.S. Afonin, PRC 83, 048202 (2011))

$$\Pi_V(-q^2) = c^2 \frac{e^{-|a|\Lambda_{\text{cut}}^2}}{2} \frac{U(1 - q^2/(4|a|), 1, |a|\Lambda_{\text{cut}}^2)}{U(-q^2/(4|a|), 0, |a|\Lambda_{\text{cut}}^2)}$$

$$\boxed{m_1^2/m_0^2 < 2}$$

$$|a|\Lambda_{\text{cut}}^2 = 1 \qquad m_n^2 = m_0^2\{1, 1.8, 2.6, 3.3, 4.1, \ldots\}$$

## Comments on the sign of the dilaton background

$$S = -\frac{c^2}{4}\int d^4x\, dz\, \sqrt{g}\, e^{-az^2} F_{MN}F^{MN}$$

Consider the background as a part of metric: $ds^2 = \frac{R^2}{z^2} e^{2A(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$

Sonnenschein criterion for the confinement: $\partial_z (g_{00})|_{z=z_0} = 0, \qquad g_{00}|_{z=z_0} \neq 0$

Consider $2A(z) = \pm\kappa^2 z^2$

For "+" there is the minimum at $z_0 = 1/\sqrt{2}\kappa.$

Qualitatively: consider an object of mass ***m*** in the 5D AdS that falls to the infrared region by the effects of gravity. Its gravitational potential energy is

$$V = mc^2\sqrt{g_{00}} = mc^2 R\,\frac{e^A(z)}{z}$$

For the positive background the potential has an absolute minimum at $\bar{z} = 1/\kappa$

$\Longrightarrow$ the potential confines any object to distances $\langle z\rangle \sim 1/\kappa$

# Soft-wall model and the chiral symmetry breaking (CSB)

Add the term $$S_{\rm CSB} = \int d^4x\, dz \sqrt{g}\, e^{-\Lambda^2 z^2} \left( |\partial_M X|^2 + \frac{3}{R^2}|X|^2 \right)$$

The scalar field corresponds to the operator $\bar{q}q$

Reminder: $$m_5^2 R^2 = (\Delta - J)(\Delta + J - 4)$$

$$\Phi(x,z)_{z\to 0} = z^{4-\Delta}\Phi_0(x) + z^{\Delta}\frac{\langle O(x)\rangle}{2\Delta - 4}$$

In the given case $\Delta = 3$

According to the holographic prescriptions: $X(z)_{z\to 0} \sim Mz + \Sigma z^3$

quark condensate

bare quark mass

There is only one solution bounded as $z \to \infty$ $$X(z) = zU(\tfrac{1}{2}, 0, \Lambda^2 z^2)$$

Thus, the quark condensate and quark condensate turn out to be related in contradiction with QCD. If we change the sign of background they are independent quantities.

# CONCLUSION

**Chiral symmetry breaking in the soft-wall holographic model in the first approximation amounts to change the sign of the dilaton background in the axial-vector sector**